Reduce It!

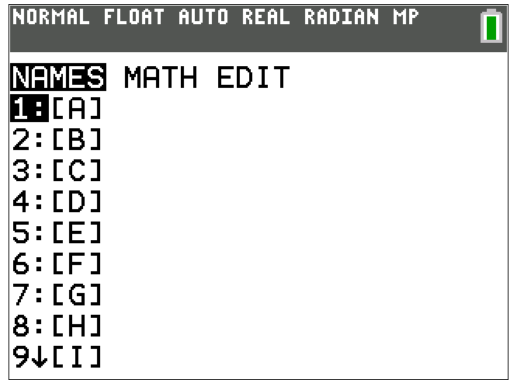
Name ______________________

Class ______________________

In this activity, you will:

- *Solve systems of equations by writing the augmented matrices in reduced row-echelon form*

Use this document as a reference and to record your answers. **Screen captures are from the TI-84 Plus CE.*



Problem 1 – Augmented matrices and reduced row-echelon form

You have already learned how to solve systems of equations such as the one to the right by graphing and using elimination and substitution.

$$\begin{aligned} 2x + 3y &= 5 \\ 5x - 4y &= -22 \end{aligned}$$

But what about larger systems like this one?
Surely you could solve this system by elimination, but what if the system had six equations in six unknowns?

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

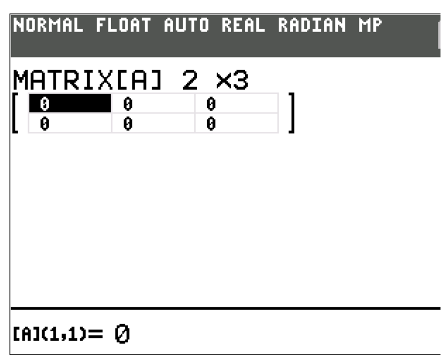
We'll explore how to solve larger systems by first solving the 2 × 2 system. The first step is to write an augmented matrix.

In an **augmented matrix**, each row represents an equation of the system (omitting the variables). Each column represents the coefficients of a specific variable, with the last column being the constant terms.

- Write the augmented matrix for the system $\begin{cases} 2x + 3y = 5 \\ 5x - 4y = -22 \end{cases}$ below.

$$\begin{array}{r c} & \begin{array}{ccc} x \text{ coeff} & y \text{ coeff} & \text{const} \end{array} \\ \begin{array}{r} \text{Eqn 1} \rightarrow \\ \text{Eqn 2} \rightarrow \end{array} & \left[\begin{array}{cc:c} \square & \square & \square \\ \square & \square & \square \end{array}\right] \end{array}$$

Define this to be matrix *A*. To do this press `   i   and arrow to EDIT. Select matrix [A], change the numbers in the top right of the screen to 2 × 3, and then enter the numbers of the matrix.



Now you will use elementary row operations to reduce the matrix in a manner similar to using elimination.

Elementary row operations performed on an augmented matrix yield an augmented matrix of an equivalent system.

The elementary row operations are:

- interchange any two rows
- multiply a row by a nonzero constant
- add a multiple of a row to another row

The goal of using these elementary row operations on an augmented matrix is to rewrite the matrix in its equivalent, reduced row-echelon form.

A matrix is in **reduced row-echelon form** if all of the following hold:

- All zero rows (if any) are at the bottom.
- The first nonzero entry in any nonzero row is a 1 (called a leading 1).
- Columns containing a leading 1 have zeros for all other entries.
- Each leading 1 appears to the right of leading 1s in rows above it.

- Use elementary row operations as described below to write matrix *A* in reduced row-echelon form.

$$A = \left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$$

a. $\frac{1}{2}r_1 \rightarrow r_1$ $\quad \left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$

b. $-5r_1 + r_2 \rightarrow r_2$ $\quad \left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$

c. $-\frac{2}{23}r_2 \rightarrow r_2$ $\quad \left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$

d. $-\frac{3}{2}r_2 + r_1 \rightarrow r_1$ $\quad \left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$

To perform elementary row operations on your calculator, use the following commands from the Matrix > MATH menu. The arguments are given in parentheses.

rowSwap(matrix, row#, row#)

..**row**(value, matrix, row#)

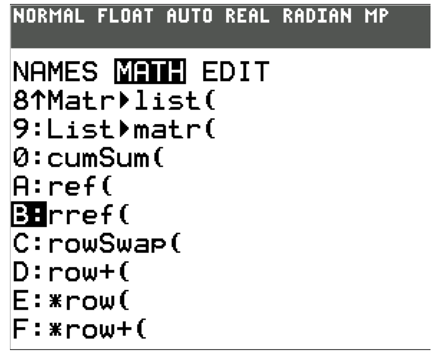
..**row+**(value, matrix, row#, row#)

Note: When you are performing subsequent row operations, use **Ans** as the matrix.)

You can check your answer using the **rref** command, which returns the reduced row-echelon form for a given matrix.

- Use this command on matrix *A*.

$$\textbf{rref}(A)=\left[\begin{array}{cc|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$$



From the reduced row-echelon form, you can easily extract the solution to the system. Since the first column represents the coefficients of x and the second column the coefficients of y, this new equivalent system is simply $x = -2$ and $y = 3$, which is the solution to our system.

- What would the reduced row-echelon form of an augmented matrix for a system with infinitely many solutions look like?

- for a system with no solutions?

Problem 2 – A 3 × 3 system

The **rref** command is very helpful when solving larger systems, but you should still know how to reduce augmented matrices yourself.

- Try it with the 3 × 3 system shown to the right. Define the augmented matrix as [A], and use the calculator to perform the elementary row operations. Check your answer using the **rref** command.

$$x - 2y + 3z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + 5z = 17$$

$$A = \left[\begin{array}{ccc|c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array}\right] \qquad \textbf{rref}(A) = \left[\begin{array}{ccc|c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array}\right]$$

Solution to system

$x =$ ______

$y =$ ______

$z =$ ______

Problem 3 – Larger systems

- For this 4 × 4 system, write the augmented matrix and solve using the **rref** command.

$$w - 2x + 2y + z = 1$$
$$3w - 5x + 6y + 3z = -1$$
$$-2w + 4x - 3y - 2z = 5$$
$$3w - 5x + y + 4z = -3$$

$$A = \left[\begin{array}{cccc|c} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{array}\right] \qquad \textbf{rref}(A) = \left[\begin{array}{cccc|c} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{array}\right]$$

Solution to system

$w =$ ______

$x =$ ______

$y =$ ______

$z =$ ______

Problem 4 – Curve fitting

Use this method to find an equation of the form $y = ax^3 + bx^2 + cx + d$ that passes through the points (–2, –37), (–1, –11), (0, –5), and (2, 19).

First, we need to generate a system of equations.
Substitute the first point for (x, y). This results in:

$$-37 = a(-2)^3 + b(-2)^2 + c(-2) + d \quad \rightarrow \quad -8a + 4b - 2c + d = -37$$

Do the same for each of the three remaining points and record the resulting system of equations. Then solve using the **rref** command.

System of equations:

$$A = \left[\begin{array}{cccc:c} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{array}\right] \qquad \textbf{rref}(A) = \left[\begin{array}{cccc:c} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{array}\right]$$

Solution to system

$a =$ ______
$b =$ ______
$c =$ ______
$d =$ ______

- What is the equation?

Exercises

Solve each system.

1. $x - 3y + z = 1$
$2x - y - 2z = 2$
$x + 2y - 3z = -1$

2. $2x + 4y + z = 1$
$x - 2y - 3z = 2$
$x + y - z = -1$

3. $x + 2y - 7z = -4$
$2x + y + z = 13$
$3x + 9y - 36z = -33$

4. The height of an object thrown into the air is determined by the equation $h = \frac{1}{2}at^2 + v_0t + s_0$ where a is the acceleration due to gravity, v_0 is the initial vertical velocity, t is the time in seconds, and s_0 is the initial height.

Using the following (*time, height*) data to determine a, v_0, and s_0 for this equation.
(1, 48), (2, 64), (3, 48)

