NUMB3RS Activity: It All Started with a Pair of Rabbits
Episode: "Sabotage"

Topic: Fibonacci Numbers **Grade Level**: 9 - 12
Objective: To identify the Fibonacci numbers and explore some of their applications
Time: 15 - 20 minutes
Materials: a scientific or graphing calculator, a pinecone, markers

Introduction

Charlie mentions the Fibonacci numbers in this episode in an attempt to explain the importance and beauty of mathematics found everywhere in nature. Fibonacci was a nickname given to Leonardo of Pisa, an Italian mathematician who lived approximately 1170–1250. He is responsible for introducing the Hindu-Arabic number system and Arabic numerals to Europe. In his book, *Liber abaci*, Fibonacci posed a seemingly straightforward problem about the number of rabbits produced in one year given a specific breeding pattern. The solution to the problem gave rise to what we now call Fibonacci numbers.

From this simple beginning eight centuries ago, this pattern of numbers continues to astonish and delight both novice and experienced mathematicians. The Fibonacci numbers appear in the rhythm of music and poetry, in the dimensions of works of art and architecture, and in the growth patterns of plants and animals. The ever-present nature of the Fibonacci numbers provides opportunities for interesting investigations in any mathematics course.

Discuss with Students

One of the largest Fibonacci sites on the Internet is **www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fib.html**. Look there to find applications to various levels of mathematics. You will find connections to geometry, prime numbers, factoring, combinatorics, Pascal's triangle, and more. Ideas found at this site will enable a teacher to connect the Fibonacci numbers with the content of the course students are taking.

The problems in this activity are intended for a general audience. They can be adapted to meet the interests and skills level of any students. For example, students with technology skills can write a program or spreadsheet to generate the answers to the table problems. Students who have difficulty visualizing patterns might use counters to display the values in the first table.

Many students would benefit from an active investigation of the Fibonacci numbers such as acting out the puzzle "Leonardo's Leap" which can be found on the Fibonacci puzzle page (**http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html**). In this activity students will show all possible ways to climb stairs by taking either one step at a time or two steps at a time (leap). Each student should demonstrate a different case, and no two students should demonstrate the same case. To start, one student will show that there is only one way to get to the first step, by taking one step. The first student climbs one step and stays there. To get to the second step there are two options (step-step, leap). Have two different students demonstrate these two options and remain standing on the second step. Have three students demonstrate the three different ways to get to the third step (step-step-step, leap-step, step-leap) and remain there. Continue this process with students remaining on their final step. If the teacher remains standing

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at the base of the steps (only one way to get there) then the pattern will be the Fibonacci numbers. Take a Fibonacci class picture and count the number of students on each step.

Student Page Answers:
1. *Column should be 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233* **2.** *Column entries 1, 2, 1.5, 1.667, 1.6, 1.625, 1.615, 1.619, 1.618, 1.618, 1.618, 1.618* **3.** *Answers could vary but the numbers will alternate between being above 1.61803… and below it with the limit being 1.61803…which rounds to 1.618.* **4.** *Starting with the 9th term all terms round to 1.618.*

5.

$$\frac{1}{x} = \frac{x}{x+1}$$

$$x^2 = x+1$$

$$x^2 - x - 1 = 0$$

$$x = \frac{1+\sqrt{5}}{2}$$

$$x \approx 1.618$$

6. *They are the same.*
7. *They are 2 consecutive Fibonacci numbers (which two would depend on the type of cone).*
8.

```
F   M   F   M   F   F   M   F   M   F   M   F   F      13
|   \   /   \   /   |   \   /   \   /   \   /   |
M     F       F     M     F       F       F     M       8
\     /       \     /     |       |       \     /
    F             F       M       M           F         5
    |             \       /       \         /
    M                 F                 F               3
      \             /                   |
              F                         M               2
              \                         /
                          F                             1
                          |
                          M                             1
```

The order in which the students list M and F could differ, but the tree should be isomorphic to this one. Isomorphic trees will share the same basic structure although the order of the letters in rows may be rearranged. The number of bees in each generation is a Fibonacci number. In this particular form, if all the Ms were black and the Fs were white in the top line, then it would be the same configuration as the black and white keys on a piano keyboard. This matches the picture of a keyboard which is at the top left of this Web page: ***http://goldennumber.net/music.htm***

Bro. Patrick Carney, De Paul Catholic High School, Wayne, NJ

Name: ________________________________ Date: _______________

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Charlie mentions the Fibonacci numbers in this episode in an attempt to explain the importance and beauty of mathematics found everywhere in nature. Fibonacci was a nickname given to Leonardo of Pisa, an Italian mathematician who lived approximately 1170–1250. He is responsible for introducing the Hindu-Arabic number system and Arabic numerals to Europe. In his book, *Liber abaci*, Fibonacci posed a seemingly straightforward problem about the number of rabbits produced in one year given a specific breeding pattern. The solution to the problem gave rise to what we now call Fibonacci numbers. (For more about the solution to the rabbit problem, see the Extensions page.)

The Fibonacci numbers are a sequence where the first two terms are both 1. Subsequent terms are generated by finding the sum of the 2 previous terms. Thus, the sequence is 1, 1, 2, 3, 5, …

1. Complete the following table. (Note that $F(3)$ is the third element in the Fibonacci sequence.)

***n* = number of term**	***F*(*n*) = value of that term**
1	1
2	1
3	1 + 1 = 2
4	2 + 1 = 3
5	3 + 2 = 5
6	5 + 3 =
7	
8	
9	
10	
11	
12	
13	

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2. Using the Fibonacci numbers generated in the first table, generate a new sequence by dividing each term by the previous term. Give each value rounded to 3 decimal places.

$F(2) / F(1)$	1 / 1 = 1
$F(3) / F(2)$	2 / 1 = 2
$F(4) / F(3)$	3 / 2 = 1.5
$F(5) / F(4)$	5 / 3 = 1.667
$F(6) / F(5)$	8 / 5 = 1.6

3. Look at the numbers in the second column in question 2. What pattern(s) can you find?

4. If you were to continue this list (the number of terms getting larger and larger), what do you think would happen to the terms of the sequence?

5. The ancient Greeks thought that the most appealing rectangle was one with a proportion of $\frac{\text{width}}{\text{length}} = \frac{\text{length}}{\text{length + width}}$. Let the width of a rectangle be 1 and the length be x. Write an equation and solve for x. Approximate the answer to 3 decimal places. This number is called the Golden Ratio.

6. What observations can you make about the Golden Ratio?

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Fibonacci’s original rabbit problem made the unreasonable assumption that each pair of rabbits produces another pair of rabbits (one male, one female) each month after the second month. Many other places in nature have a connection to the Fibonacci sequence without the need for such assumptions.

7. Find a pinecone. Look at it from the point where it was connected to the tree. From that point, there are spirals in both the clockwise and counterclockwise directions. Count the number of spirals in each of those directions. (It may be easier if you color the spirals as you count them. Use different colors for the clockwise and counterclockwise spirals.) What do you notice?

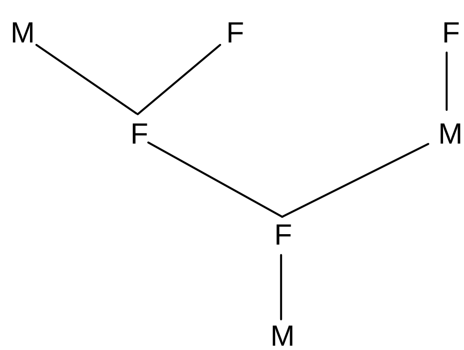
8. A male bee is born from an unfertilized egg. That means he has a mother but no father. The female bee is born from a fertilized egg, which means she has a mother and a father. Create the family tree started below for the previous 6 generations for the male bee at the bottom of the family tree below. What do you notice about the number of bees in each generation?

Male bees are represented by M and female bees are represented by F.

__ bees

__ bees

__ bees



3 bees

2 bees

1 bee

1 bee

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

The description of the Fibonacci sequence from the student pages is called a recursive definition for the Fibonacci numbers, where each term after the first two is defined using the preceding terms. That is $F_1 = 1$, $F_2 = 1$, and for $n > 2$, $F_n = F_{n-1} + F_{n-2}$. There is also an explicit formula that will yield the *n*th term directly. Try to derive it or find it by research (Hint: the $\sqrt{5}$ from the Golden Ratio will appear).

There are many connections between Fibonacci numbers and different types of plants and animals. A related research project (perhaps for a science fair) might investigate if there is a DNA sequence that the plants and animals with Fibonacci connections share with each other, but not with other organisms.

Even though Fibonacci posed his first problem about eight centuries ago, it is a remarkable source of inspiration even for today's research mathematicians. Find *The Fibonacci Quarterly,* a mathematical journal published every three months that has current research related to this famed sequence. What questions are mathematicians currently investigating?

Fibonacci made an even more profound contribution to mathematics when, after extensive travels, he brought the Hindu-Arabic number system to the West. This is the place value system we currently use. Before researching its evolution, try performing some multiplication using the Roman numeral system commonly used prior to Fibonacci's introduction of Arabic numbers.

Additional Resources

The following Web site contains a variety of connection to the Fibonacci numbers. It has references, puzzles, magic tricks, and applications which have connections to many areas. Explore the topics you find most interesting. In particular, read the rabbit problem that began it all. **http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibmaths.html**

For connections to music see **http://goldennumber.net/music.htm** or **http://techcenter.davidson.k12.nc.us/Group2/music.htm** where you can hear Fibonacci music.

If you are interested in applications to the arts and architecture go to: **http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibInArt.html**

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