12. Polor curves with TI InterActive!

introduction

This file contains a teaching idea for studying polar curves with TI InterActive!. It illustrates that the use of TI InterActive! is valuable in discovering mathematics, and shows a way of resequencing mathematics teaching. Polar curves is an exciting subject in math that can be animated in a relatively simple manner using technological tools. These tools also contribute to the development of students' understanding .

assignment

Assignment

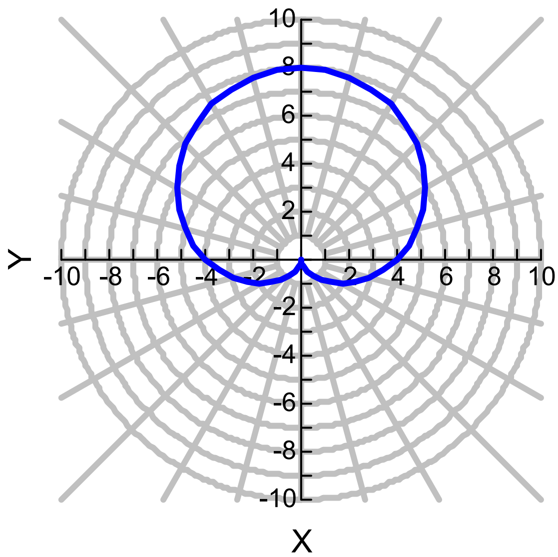
The following assignment stems from a recent national examination in Norway. We will use this to illustrate how TI InterActive! can be used to solve problems.

The curve to $r = a + a \cdot \sin(\theta)$ is called a Cardioide due to its heart-shaped form.

This assignment concerns the curve given by $r = 4 + 4 \cdot \sin(\theta)$ for $\theta \in [0, 2\pi[$.

a. Draw the curve in a polar coordinate system.

The cardioid in polar coordinates



b. Read the coordinates of the intersection points of the curve and the coordinate axes. Then calculate the coordinates.

Reading from the curve with TRACE: $\theta = 0 \Rightarrow r = 4$

$$\theta = \frac{\pi}{2} \Rightarrow r = 8$$

$$\theta = \pi \Rightarrow r = 4$$

$$\theta = \frac{3\pi}{2} \Rightarrow r = 0$$

Using calculation: $\theta = 0 \quad \Rightarrow \quad 4+4 * \sin(0) = 4$

$$\theta = \frac{\pi}{2} \quad \Rightarrow \quad 4 + 4 \cdot \sin\left(\frac{\pi}{2}\right) = 8$$

$$\theta = \pi \quad \Rightarrow \quad 4 + 4 \cdot \sin(\pi) = 4$$

$$\theta = \frac{3\pi}{2} \quad \Rightarrow \quad 4 + 4 \cdot \sin\left(\frac{3 \cdot \pi}{2}\right) = 0$$

c) Find the area defined by the curve.

The radius vector runs over a complete circulation [CYCLE or interval?], so the area is

$$\frac{1}{2} \cdot \int_0^{(2 \cdot \pi)} r^2 \, d\theta = 24 \cdot \pi$$

$$24\pi \cong 75.3982$$

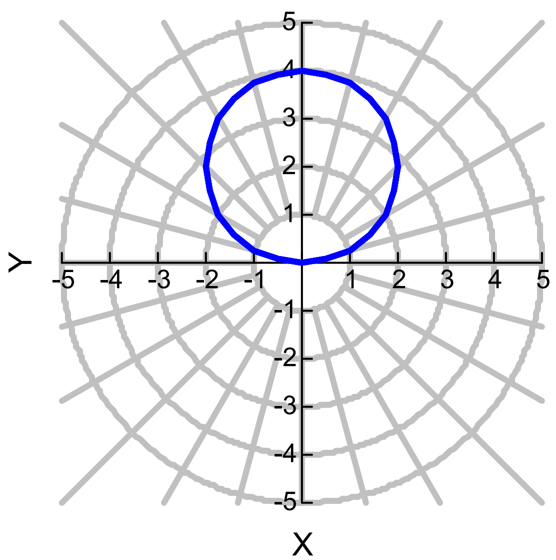
variation

Variation

What if we drop the first *a* in $r = a + a \cdot \sin(\theta)$ and study the curve given by $r = 4 \cdot \sin(\theta)$ for $\theta \in [0, 2\pi[$?

Once more, we start with the graph:

A circle in polar coordinates



The curve seems to be a circle with centre (0, 2) in rectangular coordinates and radius 2. Can we prove that this is indeed the case?

The equation of the circle is $x^2 + (y-2)^2 = 2^2$. In this case, $x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$.

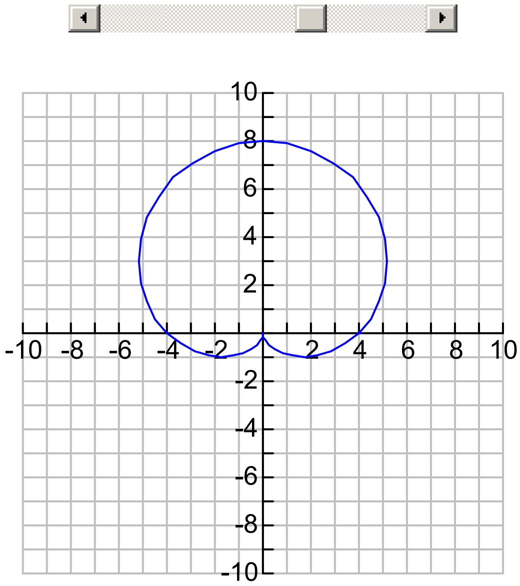
Substitution into the equation gives: $x^2 + (y-2)^2 = 2^2 \mid x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$.

exploration

Exploration

Now what if we consider the cardioid as a member of the family of curves given by $s(\theta) := 4 + a \cdot \sin(\theta)$?

Using the slider bar we can investigate this family as the value of the parameter *a* changes.

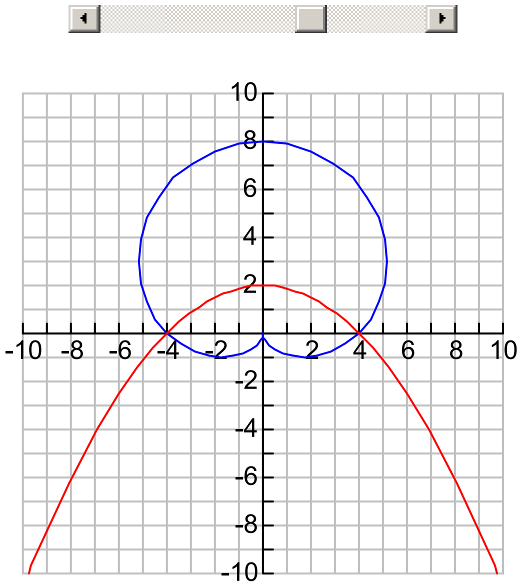


extension

Extension

Another fascinating activity is to consider the inverses of these curves: define $v(\theta) = \dfrac{16}{s(\theta)}$.

Once more, the slider bar provides insight into the dynamics:



It seems that the inverses are the conic sections; ellipse, parabola and hyperbola.

How can we prove this using TII?

As an example, we consider the case $a := 4$.

The graph suggests that the inverse is a parabola with equation $y = 2 - \frac{x^2}{8}$.

In that case, $v(\theta) = \frac{4}{\sin(\theta) + 1}$.

As a consequence, $v(\theta)\cdot\cos(\theta) = \frac{4\cdot\cos(\theta)}{\sin(\theta) + 1}$ and $v(\theta)\cdot\sin(\theta) = \frac{4\cdot\sin(\theta)}{\sin(\theta) + 1}$.

Substitution gives:

$$y = 2 - \frac{x^2}{8} \mid x = \frac{4\cdot\cos(\theta)}{\sin(\theta) + 1} \text{ and } y = \frac{4\cdot\sin(\theta)}{\sin(\theta) + 1}$$

$$\frac{4\cdot\sin(\theta)}{\sin(\theta) + 1} = \frac{-2\cdot\left((\cos(\theta))^2 - (\sin(\theta) + 1)^2\right)}{(\sin(\theta) + 1)^2}.$$

And this is true for all values of θ: $\text{solve}(ans, \theta)$.