Name ______________________________

Date ______________________________

EXPLORATIONS

ACTIVITY 10

Functioning on Your Own

What do you do when the Voyage™ 200 PLT does not have a function you want?

Create the function that you want!

In this exercise, you will learn to create and use functions. You'll start by creating a function with multiple arguments. From there, you'll create a function to return the derivative of an implicitly defined function. Finally, you'll use the Function Editor to create a function useful for solving related rate problems.

Instructions

Part A—Defining Functions with Multiple Arguments

1. Press [ON] to turn on the Voyage™ 200 PLT. To reset to the default settings, press [2nd] [MEM] [F1], select 1:RAM, select 2:Default, and then press [ENTER] [ENTER].

   Press [◆] [CALC HOME] to access the Home screen. Delete all values of one-character variables by pressing [F6], selecting 1:Clear a–z, and pressing [ENTER].

   On the Home screen, clear the entry line by pressing [CLEAR] once or twice. Clear the History area above the entry line by pressing [F1] 8.

2. Functions sometimes require more than one argument. Suppose that you wish to solve the equation $\log_x 9.84 = 3.29$.

   Define a function for a logarithm to an arbitrary base b as follows.

   Press **DEFINE LOGB** [(] **B** [,] **U** [)] [=] [LN] **U** [)] [÷] [LN] **B** [)] [ENTER].

   To solve the equation, press **SOLVE** [(] **LOGB** [(] **X** [,] **9.84** [)] [=] **3.29** [,] **X** [)] [ENTER].

   $x =$ ______________________________

3. To verify that your solution is correct, press [2nd] [ANS] [^] **3.29** [ENTER].

   Your answer should be 9.84 because $\log_x 9.84 = 3.29 \leftrightarrow x^{3.29} = 9.84$.

Part B—Defining an Implicit Differentiation Function

A function can return more than a number. In fact, you can define functions to return lists, matrices, expressions, and even equations.

In calculus, sometimes you need to find derivatives of functions defined implicitly. This process is called *implicit differentiation* and is demonstrated in the following problem.

Find the equation of a line tangent to $x^3 + y^3 = 35$ at the point (2, 3). By implicit differentiation, you obtain

$$m = \frac{dy}{dx}\Big|_{(2,3)} = \frac{-x^2}{y^2}\Big|_{(2,3)} = -\frac{4}{9}.$$

Thus the equation of the tangent line is

$$y - 3 = -\frac{4}{9}(x - 2).$$

Is there a way to take the derivative implicitly with the Voyage™ 200 PLT? The answer is yes. This is because the differential of a function in x and y of the form $F(x,y) = constant$ is given by

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 0,$$

which can be solved for $\frac{dy}{dx} = -\frac{\partial F/\partial x}{\partial F/\partial y}$.

Do not be concerned if this expression looks unfamiliar. It justifies the result that always occurs with implicit differentiation. It is essentially the same process employed in solving a related rate problem, where both x and y are considered to be functions of t; that is,

$$\frac{dF}{dt} = \frac{dF}{dx} \cdot \frac{dx}{dt} + \frac{dF}{dy} \cdot \frac{dy}{dt} = 0.$$

Solve for $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ to obtain the same result.

Functioning on Your Own *(Continued)*

1. Accordingly, to define a function called *implicit*, press [F4] and select 1:Define. Now press

   IMPLICIT [(] **EXPRESS** [,] **INDVAR** [,] **DEPVAR** [)] [=] [(-)] [2nd] [d] **EXPRESS** [,] **INDVAR** [)] [÷] [2nd] [d] **EXPRESS** [,] **DEPVAR** [)] [ENTER].

   Use this function to find the implicit derivative of $x^3 + y^3 = 35$ by pressing **IMPLICIT** [(] **X** [^] **3** [+] **Y** [^] **3** [,] **X** [,] **Y** [)] [ENTER].

   implicit derivative = ______________________________

2. Evaluate the derivative at (2, 3), and store it as m by retrieving it from the history area. Press [▲] [ENTER] [2nd] [|] **X** [=] **2 AND Y** [=] **3** [STO▸] **M** [ENTER]. ([2nd] **K** or [|] is pronounced "with".)

   m = ______________________________

3. To solve the equation, press **Y** [−] **3** [=] **M** [×] [(] **X** [−] **2** [)] [ENTER].

   equation = ______________________________

4. Now use the function *implicit* to find *dy/dx*, and write the equation of the tangent line at the given point of the following curves:

curve and point	tangent line at point
$x^2y + xy^2 = 12$ at (-4,1)	
$x^2 + y^2 = 25$ at (3,4)	
x = cos (y) at ($\sqrt{2}/2, \pi/4$) *(See Hint.)*	

Hint: *Set one side equal to zero first.*

Implicit operates as a function because it requires arguments and returns a unique response—an expression. Functions on the Voyage™ 200 PLT may operate as parts of larger expressions.

Functioning on Your Own *(Continued)*

Part C—Defining a Total Differentiation Function

1. Our last example will do even more. It will take an expression as an input and return a new expression, the total differential, with created variables such as dx and dy.

   On the Home screen, press [F4] and select 4:DelVar. Then delete x, dx, y, and dy by pressing **X** [,] **DX** [,] **Y** [,] **DY** [ENTER].

2. Now solve the following problem by hand.

   Two ships sail from a harbor, one traveling east at 24 miles per hour, and the other traveling north at 18 miles per hour. How fast is the distance between them changing when the east-bound ship is 50 miles from the harbor and the north-bound ship is 120 miles from the harbor?

   Let
   x = the distance the east-bound ship has traveled from the harbor at time t.
   y = the distance the north-bound ship has traveled at time t.
   s = the distance between the two ships at time t.

   Then the relation among the distances is $s^2 = x^2 + y^2$. Because all three variables are functions of time, take the derivative of both sides of the equation with respect to t.

   Your answer is the equation of related rates.

   equation = ______________________________

   Substitute the values $x = 50$ and $y = 120$ in the original relation, and solve for s.

   $s =$ ______________________________

   Substitute these values plus $\frac{dx}{dt} = 24$ and $\frac{dy}{dt} = 18$ into the equation of related rates. Solve for $\frac{ds}{dt}$.

   $\frac{ds}{dt} =$ ______________________________

Functioning on Your Own *(Continued)*

3. Now, solve the problem in step 2 using the Voyage™ 200 PLT.

   To create a new function called *relrates* using the Program Editor, press [APPS] and select Program Editor. Then select 3:New.

   Because you want a function and not a program, press ▶ and select 2:Function. Then press ▼ ▼ to move to Variable. Type **RELRATES** and press [ENTER]. You are now in the Program Editor.

   Type the following function lines into the Voyage 200.

   :relrates(relation, var1, var2)
   Relrates takes three arguments: the relation being evaluated and two variables.

   :Func
   Func must be the first line of a function defined in the Program Editor.

   :*d*(relation, var1)∗expr("d"&string(var1))
   +*d*(relation, var2)∗expr("d"&string(var2))
   This expression will be returned as the value of the function relrates.

   :EndFunc
   EndFunc must be the last line of a function defined in the Program Editor.

   When you finish entering the function lines, press [♦] [CALC HOME] to return to the Home screen.

Hints: *Rather than typing* **relrates(**, *you could go to the* VAR-LINK *menu (* [2nd] [VAR-LINK] *), move the cursor to select relrates, and press* [ENTER] *to paste it into the editor.*

To enter **&,** *press* [2nd] **H**.

To find the **string(** *and* **expr(** *commands, press* [2nd] [MATH] *and select* C:String.

3. (continued)
   When you solved by hand in step 2 and found the equation relating the rates

   $\frac{dx}{dt}$ and $\frac{dy}{dt}$, you should have obtained

   $2s \cdot \frac{ds}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$.

   With the Voyage™ 200 PLT, "/" cannot be used in a variable name. Therefore, in using relrates, instead of *dx/dt*, for example, only *dx* is returned.

   To define the variable *s*, press
   DEFINE S [=] [2nd] [√] **X** [^] **2** [+] **Y** [^] **2** [)] [ENTER].

   To define the differential *ds*, press
   DEFINE DS [=] **RELRATES** [(] **S** [,] **X** [,] **Y** [)] [ENTER].

   Evaluate *ds* for the particular values of the independent variables by pressing
   DS [2nd] [|] **X** [=] **50 AND DX** [=] **24 AND Y** [=] **120 AND DY** [=] **18** [♦] [ENTER].

   $ds = \frac{ds}{dt} =$ ______________________________

Extra Practice

Try using the function *relrates* with your Voyage™ 200 PLT to solve the following related rates problems.

__Hint:__ Relrates takes three arguments: an expression with two independent variables (relation), and the two variables (var1 and var2). Because the height of the top of the ladder (y) can be expressed as a function of only one independent variable—the distance of the bottom of the ladder from the wall (x)—for var2, type in a "dummy" variable that has not been defined; for example, type __dummy__.

1. An 18-foot ladder leans against the wall. Its base starts to slide away. When the base is 10 feet away, it is moving at 6 feet per second. How fast is the top of the ladder against the wall falling at that moment?

    ____

    ____

2. Sand pours onto a conical sand pile at the constant rate of 10 ft^3 per minute. If the height of the pile is always one-half the diameter of the base, how fast is the height changing when the pile is 6 feet high?

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    ____