



Math Objectives

- Students will identify the sample space of an experiment involving repeated independent trials.
- Students will calculate the probability of multiple independent events occurring.
- Students will gain understanding of the distinction of the relationship between theoretical and experimental probability.
- CCSS Mathematical Practice: Reason abstractly and quantitatively.
- CCSS Mathematical Practice: Look for and express regularity in repeated reasoning.

Vocabulary

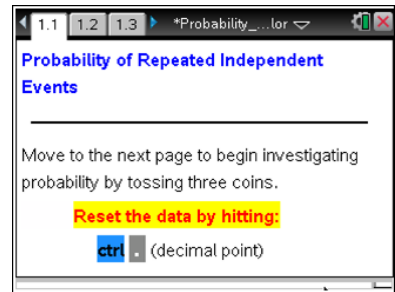
- experiment
- outcomes
- sample space
- event
- independent event
- dependent event
- probability tree diagram

About the Lesson

- This lesson involves an investigation of probability by simulating tossing a coin three times.
- As a result, students will:
 - Understand probability of specific outcomes of repeated independent trials.
 - Understand using a probability tree diagram to determine probability.

TI-Nspire™ Navigator™ System

- Use File Transfer to share the .tns file.
- Use Screen Capture or Live Presenter to monitor student progress or have them share their investigations.
- Use Quick Poll to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing **ctrl** **G**.

Lesson Materials:

Student Activity
 Probability_of_Repeated_Independent_Events_Student.pdf

Probability_of_Repeated_Independent_Events_Student.doc

TI-Nspire document
 Probability_of_Repeated_Independent_Events.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

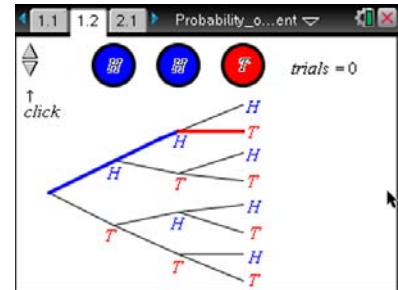


Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: *File Transfer*
See Note 1 at the end of this lesson.

1. An experiment is a process, such as tossing a fair coin three times, that gives definite results, called outcomes of the experiment. The sample space, S , of an experiment is the set of all possible outcomes. An event is any subset of the sample space.

- a. If the results of the first two coin tosses are heads, are you more likely to toss a head or a tail on your third toss? Explain.



Answer: You are equally likely to toss a head or a tail. There is a 50% probability of tossing either one.

- b. If an experiment consists of tossing a fair coin three times and recording the results in order, what is the sample space for this experiment? Use H for heads and T for tails. How many possible outcomes are there?

Answer: $S = \{HHH, HHT, HTH, HTT, TTH, THT, THH, TTT\}$. There are 8 possible outcomes in the sample space.

- c. What does it mean for a coin toss to be “fair”? Explain.

Answer: There is exactly a 50% chance of getting either heads or tails on any given toss.

Teacher Tip: A coin that is not a fair coin is called biased or unfair. This could be due to the indentions of the coin causing it to be weighted in such a way that favors one result over another. One mathematician, Persi Diaconis, has even claimed that a coin will land in the same way that it started (in your hand) about 51 out of 100 times.
(<http://news.stanford.edu/news/2004/june9/diaconis-69.html>)



- d. When the occurrence of one event does not affect the probability of another event, the events are independent. Suppose you randomly choose a ball from a bag of 10 colored balls. The first ball is not replaced. Then, you choose another ball. Are your choices independent events or dependent events?

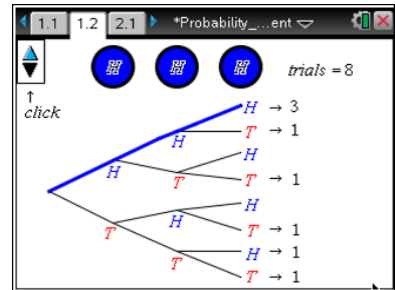
Answer: These events are dependent events. If the balls were replaced, then they would be independent events.

TI-Nspire Navigator Opportunity: Screen Capture or Live Presenter

See Note 2 at the end of this lesson.

Move to page 1.2.

2. A probability tree diagram is another way to represent the sample space of an experiment. From the left trunk, the first pair of branches represents the possible results of the first coin flip. From each of those branches, two more branches represent the possible results of the second coin flip. Finally, from each of those four branches, two more branches represent the possible results of the third coin flip.



- a. Watch the tree diagram change as you click the arrow in the top left-hand corner. Looking at the path, what is the probability of getting tails on all three tosses?

Answer: The probability of getting tails on all three tosses is $\frac{1}{8}$.

Teacher Tip: Students may have a difficult time understanding how a probability tree diagram works the first time they see it. Allow students the time to explore and see the path of the tree diagram change as they click for new trials.

- b. To find the probability of multiple independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. Here is the rule defined symbolically: $P(A \cap B) = P(A) \cdot P(B)$. Use the rule to algebraically find the probability of getting tails on all three tosses.

Answer: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$ Multiply the probability of getting tails for each of the three tosses.



- c. In 2010, the New Orleans Saints won the Super Bowl coin toss, becoming the 13th straight NFC team to win the Super Bowl coin toss. That seems highly improbable. What is the probability of correctly guessing the outcome of each fair coin toss 13 times in a row?

Answer: $\left(\frac{1}{2}\right)^{13} = \frac{1}{8,192}$

Teacher Tip: Students may try to over-complicate this problem by treating the team's call and the result of the toss as individual events. While this can be done, because the probability of a win or a loss is always 0.50, they can calculate the probability with a Win or Loss as a single event.

CCSS Mathematical Practice: Students will reason abstractly and quantitatively.

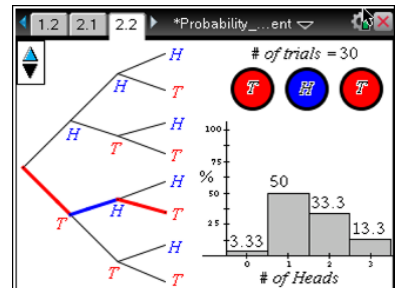
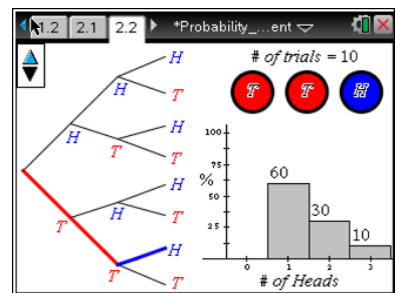
Mathematically proficient students should understand from part c that an outcome is one that is unlikely to be due to chance alone, but can only be calculated under randomness and multiplication rules.

Tech Tip: Press then to reset the number of trials.

TI-Nspire Navigator Opportunity: Screen Capture
See Note 3 at the end of this lesson.

Move to page 2.1 and then page 2.2.

3. A bar graph records the percentage of trials that have had exactly 0, 1, 2, or 3 heads showing each time the arrow in the top left-hand corner of the screen is clicked. Reset the number of trials to zero by pressing . After ten trials, record the percentages in the table below under the experimental column. Do twenty additional trials, then record your data in the appropriate column. Then, use the tree diagram to find the theoretical percentages and fill them in the correct column of the table.





Sample answer: See table below. Answers may vary for experimental data.

# of Heads Showing	Experimental (% after 10 trials)	Experimental (% after 30 trials)	Theoretical (write as a %)
0	0%	10%	$\frac{1}{8} = 0.125 = 12.5\%$
1	50%	43.3%	$\frac{3}{8} = 0.375 = 37.5\%$
2	40%	43.3%	$\frac{3}{8} = 0.375 = 37.5\%$
3	10%	3.33%	$\frac{1}{8} = 0.125 = 12.5\%$

- a. The event, “getting one head showing,” is a compound event. This indicates that it includes more than one outcome, and the probability that it occurs is the sum of the probabilities of the three separate outcomes. What are all of the possible outcomes that show exactly one head? Use the outcomes to find the theoretical probability.

Answer: *HTT, THT, and TTH*. The probability of each of these outcomes is $\frac{1}{8}$. To find the

probability of the compound event, just add these together: $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$.

- b. Do your percentages in the experimental columns match the percentages in the theoretical column? If not, why?

Answer: The experimental columns will not match the theoretical exactly. They may be somewhat close, or not.

Teacher Tip: Students may have a difficult time understanding why the experimental column and the theoretical column don’t match. The experimental results will vary because they are the result of random flips of the coin. The theoretical outcomes are the expected value, which is like an educated guess as to what the outcomes of the experiment will be. Try to get your students to verbalize the reason for this disparity.

CCSS Mathematical Practice: Students will look for and express regularity in repeated reasoning.

Mathematically proficient students should understand that random processes as done by this model are simulating experimental data. Using addition and multiplication rules, students should be able to describe



theoretical outcomes and the difference from experimental outcomes.

- c. What do you think would happen to the data if you did thousands (or even millions) of trials?

Answer: The data should approach the theoretical values (also called the expected values).

Teacher Tip: This may be a good time to introduce the Law of Large Numbers theorem. It states that the probability that the experimental average is NOT close to the theoretical will decrease as the number of trials gets larger. In other words, the result of performing the same experiment a large number of times will yield an average of the results that is increasingly likely to be close to the expected value. What this means practically is, if we continue to run repeated independent trials and keep track of the cumulative average, it becomes more and more likely that our values will be close to the theoretical (but it does not guarantee it).

TI-Nspire Navigator Opportunity: Quick Poll
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to use the rule of multiplication to calculate the probability of independent events.
- How to use a probability tree diagram to determine the probability.

TI-Nspire Navigator

Note 1

Entire Document, File Transfer: Use the file transfer to efficiently send the .tns file to the students. Using TI-Navigator will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, Screen Capture or Live Presenter: If students experience difficulty with operation of a file or a question, use the computer software or *Live Presenter* with TI Navigator. You can also use this as a way to facilitate student discussion.

Note 3

Page 2.2, Screen Capture: Particularly on page 2.2, *Screen Capture* should be used for students to compare their bar graphs with other students in the class.



Note 4

End of lesson, Quick Poll: A *Quick Poll* can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students may have.

The following are some sample questions you can use:

1. In this experiment, what is the probability of getting heads on both of your first two coin tosses?

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{3}{8}$

d. $\frac{1}{2}$

2. In this experiment, what is the probability of getting at least two tails on your three tosses?

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{3}{8}$

d. $\frac{1}{2}$

3. True/False: In this experiment, the probability of getting all tails is the same as the probability of not getting all tails.

Answer: False

Teacher Tip: Given the probability of an event (i.e. probability of getting all tails), the probability of its complement (i.e. probability of NOT getting all tails) can be found by subtracting the given probability from 1.