EXPLORATIONS™

Activity 14

Triangle Inequality Theorem

Objectives

- To investigate relationships between angle measurements and sides of a triangle
- To investigate relationships among the three sides of a triangle

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Introduction

The Triangle Inequality Theorem is an important and useful theorem in geometry. In this activity, you will explore the concepts underlying this theorem by comparing side lengths and angle measures. You will use an interactive construction to see why this theorem is true.

Part I: Investigate Angles and Sides

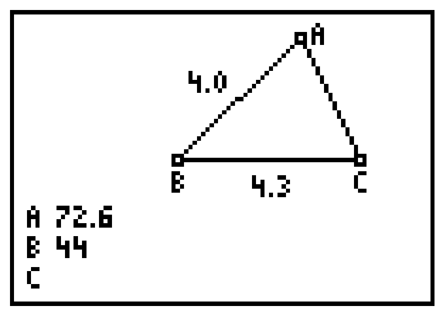
Construction

Draw a triangle and measure its angles and sides.

Draw ΔABC near the center of the screen.

Measure each of the three interior angles.

Measure the length of each of the three sides.



Note: Not all measurements are shown.

Exploration

Observe the relationships between the angle measures and the length of the side opposite the angle. Drag one of the vertices or sides of the triangle and see if the relationship remains true.

Questions and Conjectures

1. Make a conjecture about the relationship between the measure of an angle and the length of the side opposite that angle.
2. What does your conjecture above imply about triangles that have two or three sides that have the same length? Explain your reasoning and be prepared to demonstrate.

Part II: Longest and Shortest Possible Sides

Construction

Construct a special triangle as described below.

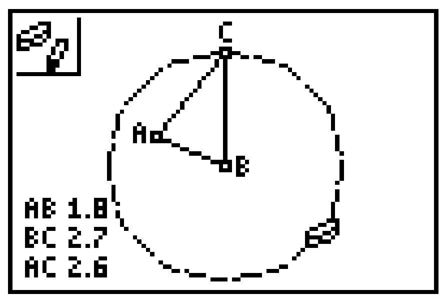
Clear the previous construction.

Draw a circle in the center of the screen. Hide the radius point of the circle.

Construct $\triangle ABC$ so that point A is inside the circle, point B is the center of the circle, and point C is a point on the circle (but not the radius point).

Measure the length of each of the three sides.

Hide the circle.



Exploration

Verify by dragging that the following are true: the length of side $\overline{BC}$ remains constant when point A is moved, the lengths of sides $\overline{AB}$ and $\overline{BC}$ remain constant when point C is moved, and the lengths of all sides change when point B is moved.

Drag the vertices of the triangle and observe any relationships that exist among the three sides of the triangle. Be sure to note the relationship when the points A, B, and C do not form a triangle.

Calculate the sum $AC + AB$ and the difference $AC - AB$. Investigate the relationship between the calculated quantities and the length of side $\overline{BC}$. Be sure to note the relationship as you drag point A.

Calculate the difference $BC - AB$ and the sum $BC + AB$. Investigate the relationship between these calculated quantities and the length of side $\overline{AC}$. Be sure to note the relationship as you drag point C.

Questions and Conjectures

1. Make conjectures about the relationships among the three sides of a triangle.
2. You are given the length of one side of a triangle. Make a conjecture about the length of the other two sides of the triangle. Explain your reasoning and be prepared to demonstrate.
3. You are given the lengths of two sides of a triangle. Make a conjecture about the length of the third side of the triangle. Explain your reasoning and be prepared to demonstrate.
4. Given three segments, under what conditions would these segments form a triangle?

Teacher Notes

Activity 14

Triangle Inequality Theorems

Objectives

- To investigate relationships between angle measurements and sides of a triangle
- To investigate relationships among the three sides of a triangle

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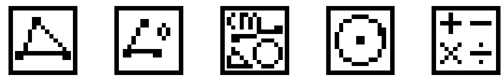
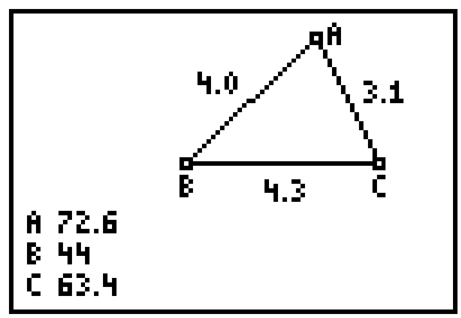


Part I: Angles and Sides

Answers to Questions and Conjectures

1. Make a conjecture about the relationship between the measure of an angle and the length of the side opposite that angle.

   Students should notice that when $\triangle ABC$ is scalene, the largest angle is opposite the longest side and the smallest angle is opposite the shortest side.



2. What does your conjecture above imply about triangles that have two or three sides that have the same length? Explain your reasoning and be prepared to demonstrate.

   When the measure of two sides of a triangle are equal, the measure of the angles opposite the equal sides are congruent. When the measure of the three sides of the triangle are equal, then the measure of each of the three angles will equal 60°. Dragging a vertex of the triangle should support these conjectures.

Part II: Longest and Shortest Possible Sides

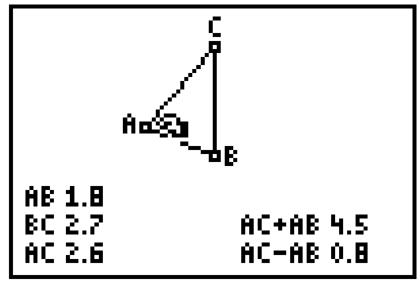
Answers to Questions and Conjectures

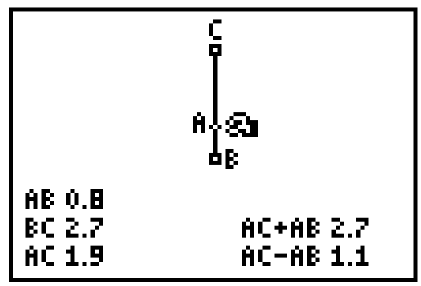
1. Make conjectures about the relationships among the three sides of a triangle.

   In general, there are no relationships among the three sides of a triangle as there are among the three angles of a triangle. The actual relationships are described in the answers that follow.

2. You are given the length of one side of a triangle. Make a conjecture about the lengths of the other two sides of the triangle. Explain your reasoning and be prepared to demonstrate.

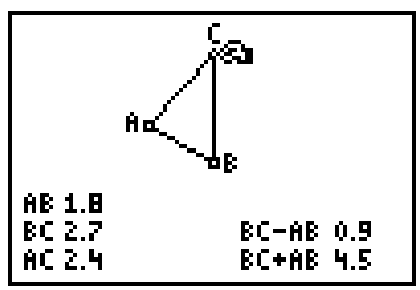
   In order for a triangle to be formed, the sum of the lengths of the other sides of the triangle must be greater than the length of the given side and the difference must be less than the length of the given side. This can be demonstrated by looking at the sum of *AC* + *AB* and the difference *AC* - *AB* and comparing these values to $\overline{BC}$ as you drag point *A*.

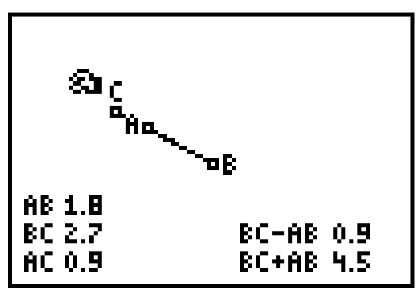




3. You are given the lengths of two sides of a triangle. Make a conjecture about the length of the third side of the triangle. Explain your reasoning and be prepared to demonstrate.

   In order for a triangle to be formed, the third side must be longer than the difference in the lengths of the given sides, but not as long as the sum of the lengths of the given sides. This can be demonstrated by looking at the difference BC - AB and the sum BC + AB and comparing this value to $\overline{AC}$ as you drag point *C*.





4. Given three segments, under what conditions would these segments form a triangle?

   Given any three segments, the sum of lengths of any two must be greater than the length of the third segment in order for a triangle to be formed. In a triangle, the sum of any two sides must be greater than the third.