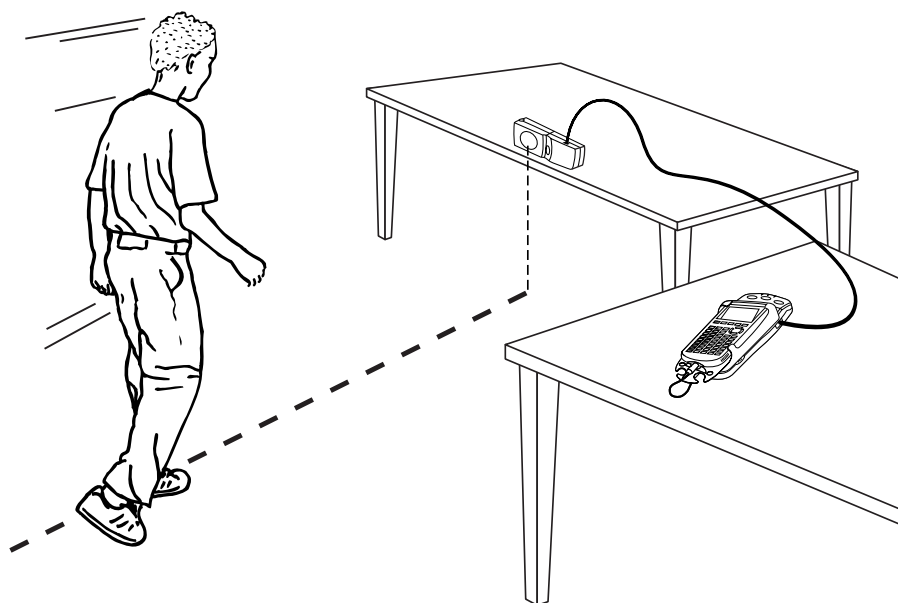


Graph It in Pieces: Piecewise Defined Functions

Graphs of real-world data cannot always be described with one simple equation. Often the graph is made up of separate pieces which together describe an event. If you move back and forth in front of a Motion Detector, your motion could be described in separate pieces which together would describe the total motion. Functions which are defined in pieces are called *piecewise-defined functions*.

In this activity, you will create a graph by moving back and forth in front of a Motion Detector. You will then describe your motion by writing a piecewise-defined function for the motion.



OBJECTIVES


- Record motion data for a walker.
- Describe segments of the walker's motion using linear functions.
- Model the motion data using a piece-wise continuous function assembled from linear functions.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
CBR 2 or Go!Motion and direct calculator cable
or Motion Detector and data-collection interface

EasyData application
meter stick
masking tape

PROCEDURE

1. Set up the Motion Detector and calculator.
 - a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown. 
 - b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Set up the Motion Detector on a table or desk. The detector should be aimed horizontally at a height of about 1.2 m. Use the meter stick and masking tape to mark $\frac{1}{2}$ meter intervals from the detector. Write the distance from the detector on each piece of masking tape.
3. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **(File)** from the Main screen, and then select **New** to reset the application.
 - c. Select **(Setup)** from the Main screen, and then select **Time Graph...**
 - d. Select **(Edit)** from the Time Graph Settings screen.
 - e. Enter **0.2** as the time between samples in seconds and select **(Next)**.
 - f. Enter **50** as the number of samples and select **(Next)**.
 - g. Select **(OK)** to return to the Main screen.
4. Practice walking in the following manner: Stand about 1.5 m from the detector. When you hear the detector start to click, stand still for about four seconds. Then start walking slowly away from the detector at a uniform rate for about three seconds. Stop and then walk slowly toward the detector until the clicking stops. Do not get any closer than 0.5 m to the detector.
5. Select **(Start)** to begin data collection. Walk as you practiced. Data collection will run for ten seconds. When data collection ends, a graph of distance versus time will appear on the screen.
6. Examine the distance versus time graph. The graph should contain a series of three nearly straight-line regions. Check with your teacher if you are not sure whether you need to repeat the data collection.

To repeat data collection, select **(Main)** and repeat Step 5.

7. Once you are satisfied with the graph, select **(Main)** to return to the Main screen. Exit EasyData by selecting **(Quit)** from the Main screen and then selecting **(OK)**.

ANALYSIS

1. Redisplay the graph outside of EasyData.
 - a. Press **(2nd)** [STAT PLOT].
 - b. Press **(ENTER)** to select Plot1 and press **(ENTER)** again to select On.
 - c. Press **(ZOOM)**.
 - d. Press **(\downarrow)** until ZoomStat is highlighted; press **(ENTER)** to display a graph with the x and y ranges set to fill the screen with data.
 - e. Press **(TRACE)** to determine the coordinates of a point on the graph using the cursor keys.

2. To create an equation that describes the motion shown in the plot, you will first need to write a linear equation for each section of the graph. Then you can put the pieces together to define a piecewise-defined function.

Your walking instructions were in three parts, each describing uniform motion. So, to model the motion graph you should use three straight-line pieces. Each one will have the linear form $y = mx + b$, each with its own slope m and y -intercept b .

Trace across the graph and find the x and y coordinates of the first point on the graph (x_1 and y_1). Trace to the end of the first section of the graph and find the coordinates of this point (x_2 and y_2). Record these two points in the Data Table on the *Data Collection and Analysis* sheet.

⇒ Answer Questions 1 and 2 on the *Data Collection and Analysis* sheet.

3. Check to see that this equation does fit the data for the first section of the graph. If the equation does not match the data for the first section of the graph, make adjustments to m or b so that it gives a better fit.

- Press Y= .
- Press CLEAR to remove any existing equation.
- Enter the equation for the segment. For example, for a slope of 3 and an intercept of 2, enter $3 \cdot X + 2$.
- Press GRAPH to see the segment and data.
- Press 2nd [QUIT] to return to the home screen.

4. You have now matched the first segment of the graph. To do the same for the second segment, first turn off the plotting of the function for the first segment.

- Press Y= .
- Move to the equation with the cursor keys.
- To turn off the plotting of the function, press ◀ until the $=$ sign is highlighted. Press ENTER to deselect the equation.
- Press 2nd [QUIT] to return to the home screen.

5. Redisplay the graph and trace across to the end of the second segment to get the coordinates of a point at the end of the segment (x_3 and y_3).

- Press GRAPH .
- Press TRACE to determine the coordinates of a point on the graph using the cursor keys.
- Press 2nd [QUIT] .

Record the point (x_3 and y_3) in the Data Table on the *Data Collection and Analysis* sheet.

6. The equation for the line matching the second segment can be determined from the points you have recorded. Since the point (x_2, y_2) is also at the beginning of the second segment, you have enough information to determine the slope of this segment.

⇒ Answer Question 3 on the *Data Collection and Analysis* sheet.

7. You still need to find the y -intercept of this segment, but you cannot just trace to the y axis as you did before because this segment does not go that far. However, using the definition of slope we can write down the point slope form of a line, or $y - y_3 = m(x - x_3)$. Here y and x are variables, m the slope, and x_3 and y_3 are the values of a point on the line.

Activity 6

⇒ Use this relation to answer Question 4 on the *Data Collection and Analysis* sheet.

8. Enter the equation for the second segment in the Y2 equation register using the same method you did in Step 3, but press \checkmark to use the Y2 line. Do not clear the Y1 equation. Observe the way the line fits the second segment of the graph, and adjust your slope and intercept values if needed.
9. Use the method from Step 4 to turn off the plotting of equation Y2.
10. Redisplay the graph and trace to the end of the third segment to read the coordinates of a point there (x_4 and y_4). Record the point (x_4 and y_4) in the Data Table.
⇒ Answer Question 5 on the *Data Collection and Analysis* sheet.
11. Enter the equation for the third line segment in the Y3 equation register, display its graph with the data, and adjust the slope and intercept if necessary. Turn off the plotting of equation Y3 when you are done.
12. So far you have created equations that fit the motion data in a piecewise manner. The next step is to combine those equations for plotting, each plotted only over the appropriate x range. Write down the x ranges over which you would like to plot each of your three equations Y1, Y2 and Y3.
13. The calculator can perform tests which will allow you to assemble the functions into one expression. The expression ($x < 1.2$) would be evaluated as 1 if x has a value less than 1.2, and otherwise would be evaluated as zero. Using this, you can force a function to be plotted only over the appropriate range. For example, $Y2^*(x < 1.2)$ would only be graphed for x less than 1.2, and would be zero everywhere else.

In the following steps you can build an expression to plot the three lines as a piecewise defined function. Where you see x_2 and x_3 substitute the x values from the Data Table. For example, if you were to use values for x_2 and x_3 of 2.2 and 4.5, the expression for equation Y4 would look something like this:

$$Y4=Y1*(x<2.2) + Y2*(x\geq 2.2 \text{ and } x\leq 4.5) + Y3*(x>4.5)$$

You can understand this expression by looking at each product separately. Only the Y1 equation will be plotted for $x < 2.2$, since the other equations are multiplied by zero. For x between 2.2 and 4.5, only Y2 will be plotted, with Y1 and Y3 being multiplied by zero.

- a. Press Y= .
- b. Move to equation Y4 with the cursor keys.
- c. Enter the following expression.

$$Y1*(X < x_2) + Y2*(X \geq x_2 \text{ and } X \leq x_3) + Y3*(X > x_3)$$

Enter the $>$ symbols by pressing 2nd [TEST] and selecting the desired symbol from the menu. Enter equation labels like Y1 by pressing VAR , pressing D to display the Y-VARS menu, and selecting 1:Function. Select the desired function name from the list. Enter the and function by selecting 2nd [TEST] and pressing D to display the LOGIC menu. Select and from the list. Remember to substitute your own values for x_2 and x_3 .

- d. Press C until the icon to the left of Y1 is blinking. Press ENTER until a bold diagonal line is shown to display the model with a thick line.
- e. Press GRAPH to see the piecewise function plotted with the motion data.

f. Press $\text{\textcircled{2nd}}$ [QUIT] to return to the home screen.

⇒ Answer Questions 6-8 on the *Data Collection and Analysis* sheet.

EXTENSION

1. José created a distance versus time graph by starting at the 2-meter mark on the floor. He walked towards the detector at 0.25 m/s for 4 seconds, stood still for 2 seconds, walked away from the detector at 0.4 m/s for 2 seconds, and then stopped for 2 seconds. Sketch a plot of José's distance versus time graph.
2. What was José's ending position?

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

x_1		y_1	
x_2		y_2	
x_3		y_3	
x_4		y_4	

QUESTIONS

1. The slope of a line is given by the formula shown below. Calculate the slope of the first line segment between the points given.

$$m = \frac{y_2 - y_1}{x_2 - x_1} =$$

2. Now determine the y-intercept of the first segment. That is easy this time, as the segment runs into the y axis. Use the y value of your first point to obtain the value for b , and write the equation for the first section of the graph below.

3. Use the method given in Question 1 to find the slope of the second segment.

$$\text{slope } m_2 =$$

4. Write the equation of the line fitting the second segment in the traditional $y = mx + b$ form.

5. Use the same slope and point-slope relations to determine the slope and intercept of a line function that fits your third segment.

$$\text{slope } m_3 =$$

$$\text{y-intercept} =$$

