Teacher Notes

Activity 8

Move My Way—A CBR Analysis of Rates of Change

Objectives

- Explore the relationship between position and velocity
- Explore the relationship between functions and their derivatives
- Connect mathematical relationships to real-world phenomena

Materials

- TI-84 Plus / TI-83 Plus
- Calculator-Based Ranger™ (CBR™)

Teaching Time

- 60 minutes

Abstract

In this activity, students will collect position data using a CBR™ data collection device in realtime. They will use the ∆**List** feature of the graphing handheld to reinforce the idea of velocity being the rate of change of position with respect to time at all points. They will then try to match a given velocity graph and sketch the corresponding position graph.

Management Tips and Hints

Prerequisites

Students should:

- have knowledge about average rates of change.
- know the definition of *derivative*.

Student Engagement

Begin by having a student walk back and forth in front of the CBR, and ask students to predict what the velocity versus time graph will look like. They should think about what velocity means as well as how positive velocity and negative velocity differ. This activity is best done with students working in cooperative groups so that they can compare answers and look for connections. A class discussion of basic concepts after the activity would enhance understanding and ensure that students made correct observations.

Evidence of Learning

Students are asked to make predictions prior to collecting data and graphing equations. This allows them to correct their own thinking. A class discussion or analysis of answered questions will demonstrate student knowledge.

Common Student Errors/Misconceptions

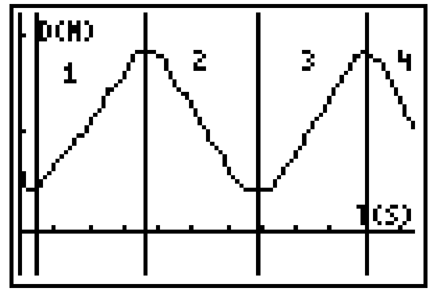
- Many students have difficulties interpreting velocity graphs for a physical situation.
- Many students confuse position graphs with velocity graphs.
- When students see a constant velocity on a velocity graph, they may mistakenly try to match it by standing still.

Extensions

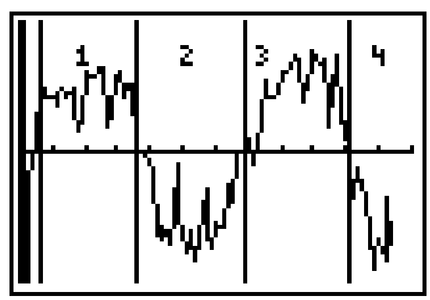
Students could develop velocity graphs for other groups to match. They could also devise a method to get the position versus time graph from the velocity graph using a Riemann Sum formula.

Activity Solutions

1. n/a
2. n/a
3. Results will vary depending on student data. A sample is shown:



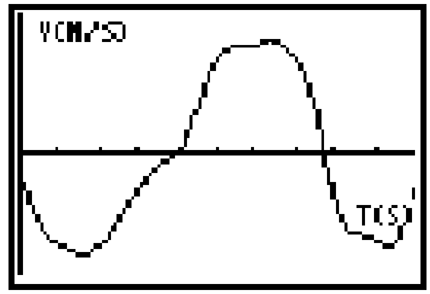
4. For the graph given, the velocity is positive for the first region, negative for the second region, positive for the third region, and negative for the last region.
5. Students should label the velocity as zero when the graph has a relative maximum or minimum.
6. Check students' graphs. The zero points on the graph above should correspond to points where the graph crossed the time axis. The velocity should be above the time axis when it is positive and below the time axis when it is negative.
7. n/a
8. The number of differences is 93. Taking the difference between items in a list will always produce a list with one fewer item.
9. n/a
10. Results will vary depending on student data. The velocity graph for the position graph above is shown:



11. On a position versus time graph, the velocity is positive when the graph is increasing.

12. On a velocity versus time graph, the velocity is positive when the graph is above the time horizontal axis.

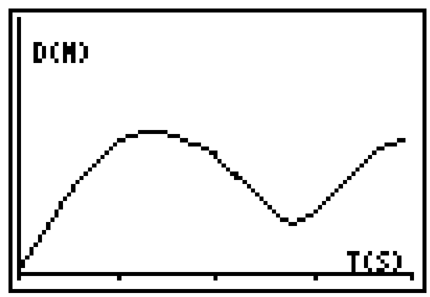
13. The graph should be similar to the one shown. It is correct as long as the graph drawn by the student is above or below the time axis for the time intervals shown. The shape above or below may vary somewhat.



14. The graph is generated by walking away from the CBR™ at a constant rate, then walking toward the CBR at a constant rate, and then walking away for a second before turning around.

15. The starting point does not affect the velocity graph because this graph only shows the rate of change.

16. The graph should show a linear section with positive slope, then a maximum, followed by a linear section with negative slope, then a minimum, then a section that increases, and a maximum at the end. It should be similar to the graph shown:



17.

When the function graph is...	The derivative graph is...
Increasing	positive, above the time axis
Decreasing	negative, below the time axis
Changing from increasing to decreasing	zero, going from positive to negative
Changing from decreasing to increasing	zero, going from negative to positive
A constant value	zero

18. Answers will vary.

EXPLORATIONS™

Activity 8

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Materials

- TI-84 Plus / TI-83 Plus
- Calculator-Based Ranger™ (CBR™)

Introduction

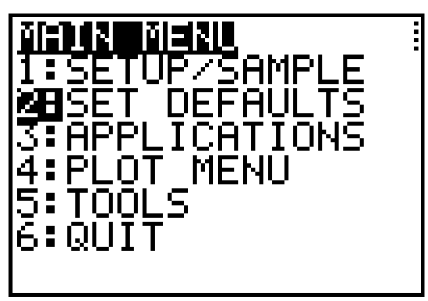
Speed and velocity are concepts that you have observed since early childhood. Because velocity is the derivative of displacement, taking a closer look at position and velocity can develop a deeper understanding of how derivatives and their functions are related.

In this activity, you will use a Calculator-Based Ranger™ (CBR™) data collection device with your graphing handheld to collect motion data with certain characteristics and use that data to draw conclusions about the relationship between position and velocity.

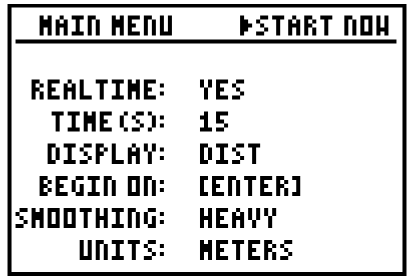
Exploration

A CBR emits an ultrasonic beam that reflects off the closest object and returns to the CBR. Depending on the speed of sound and the time it takes for the sound wave to return, the distance between the object and the CBR is calculated.

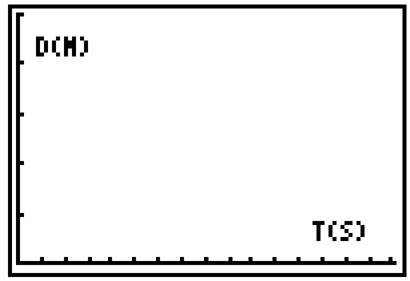
1. Begin by selecting **:CBL/CBR** from the **APPS Menu**. Press any key. Then select **3:RANGER** and press [ENTER]. Select **2:SET DEFAULTS**. The CBR will collect distance data for 15 seconds. You will see the graph as it is created and be able to analyze what type of movement causes the graph to increase or decrease.



2. Move back and forth in front of the CBR™, or point the CBR at the wall and move back and forth from the wall, changing directions at least two times. The CBR collects data representing the distance from the CBR as a function of time. This data represents the distance between either you and the CBR or the CBR and the wall, depending on your experimental setup.



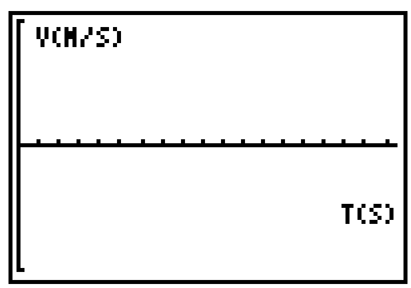
3. Make a sketch of your data on the grid shown. Write a description of the motion.



4. The velocity is considered positive when the distance from the CBR is increasing and negative when the distance from the CBR is decreasing. Identify regions on your graph where the velocity is positive and where it is negative. Clearly indicate those regions on the graph.

5. The velocity is zero when you stop, even if it is momentary. Remember that in order to change directions you had to first stop, even if you did not think you came to a complete stop. You cannot move forward and then move backward without first stopping. Label all points on your graph where the velocity is zero. What do these correspond to on the position versus time graph?

6. From your data, make a sketch of what the velocity versus time graph will look like for your walk. It is easiest to begin by plotting the zero points and then graphing the velocities as positive or negative. Do not worry about estimating the velocity values. The idea is just to sketch what the graph will look like.



7. Instantaneous velocity is the average velocity as the change in time approaches zero:

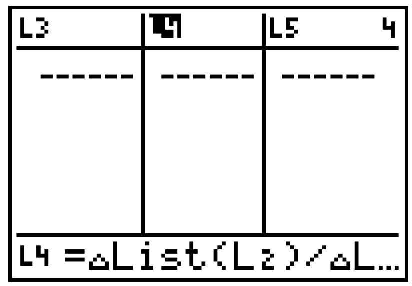
$$\left(\frac{\Delta d}{\Delta t}\right)$$

In mathematical notation:

$$v = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

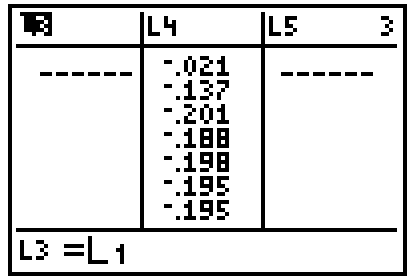
where Δd is the change in distance from the CBR™ and Δt is the elapsed time. To approximate the instantaneous velocity, you can divide the change in distance by the change in time for each set of points taken. This is the average velocity over each small time interval.

Because the time intervals are small, these values will approximate the velocity function. Select **1:Edit** in the **STAT Menu**. Move the cursor to the top of **L3** so that the **L3** is highlighted, and press [CLEAR] and [ENTER] to clear the list. Do the same for **L4**. When the lists are clear, calculate the average velocity for each time interval, and place it in **L4** by moving the cursor to the top of **L4** and selecting **7:ΔList** from the **LIST OPS Menu**. Calculate **ΔList(L2)/ΔList(L1)**. The ΔList operates by taking the difference between successive items in a list.

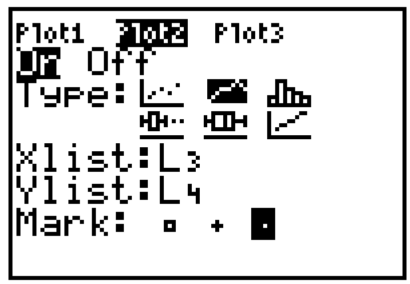


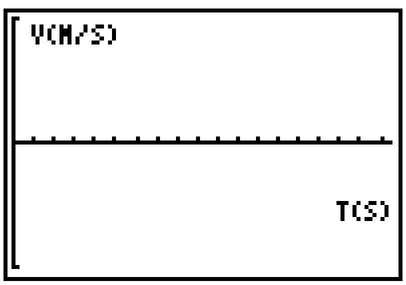
8. If the CBR collected 94 data points, how many values will be in **L4**? In other words, how many differences will be calculated? Explain your answer.

9. Because there will only be 93 values in **L4**, you will need to have a list of times with only 93 data points to plot. The actual velocity that you calculated will occur at the midpoints of all the time intervals. But because the time intervals are short, you can use the time values from **L1** and delete the first one to get a good idea of what the velocity graph looks like. Select **1:Edit** from the **STAT Menu**, and move the cursor to the top of **L3** so that the **L3** is highlighted. Input **L1** and press [ENTER] to place the values of **L1** into **L3**. Highlight the first value, and press [DEL] to delete it.



10. Turn on a stat plot for velocity as a function of time (**L3** and **L4**) as shown in the screen to the right. Sketch the actual velocity graph on the screenshot below. How does it compare with your prediction?

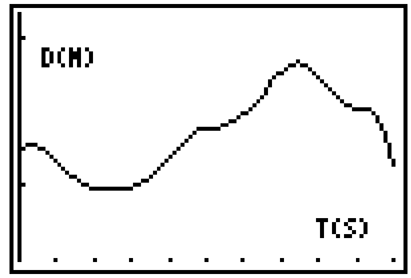


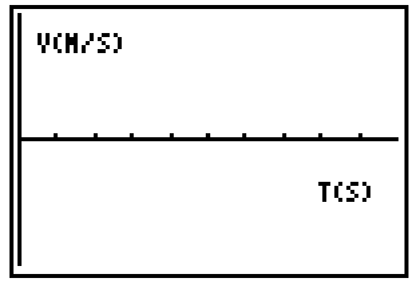


11. What does positive velocity look like on a distance versus time graph?

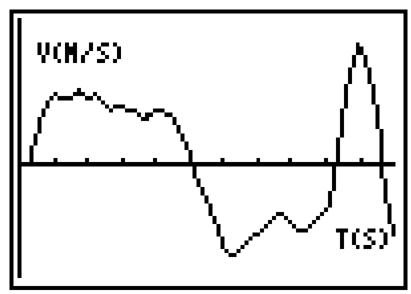
12. What does positive velocity look like on a velocity versus time graph?

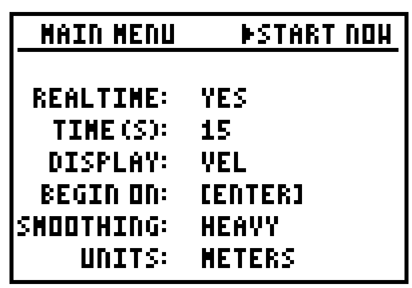
13. Make a sketch of the velocity versus time graph for the position versus time graph shown. Again, do not worry about plotting each velocity value exactly. This is just a sketch to show that you understand when the velocity is positive, negative, or zero.





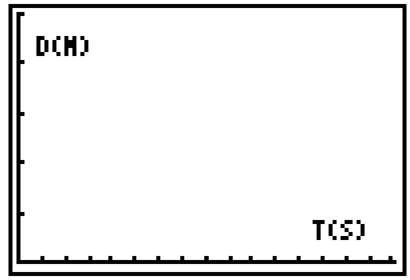
14. Use the **RANGER** application to collect velocity data so that your graph matches the one shown to the right. Your graph will not match exactly, but you should be able to approximately match each section. After executing **prgmRANGER**, select **1:SETUP/SAMPLE** and set up the collection options as shown. Be sure to set the **SMOOTHING** to **HEAVY**. Try to match the graph. Describe your motion.





15. Did your starting point affect the graph of velocity versus time? Explain.

16. Make a sketch to show what the position versus time graph would look like for the velocity graph that you matched.



17. After completing this activity, complete the chart shown:

When the function graph is...	The derivative graph is...
Increasing	
Decreasing	
Changing from increasing to decreasing	
Changing from decreasing to increasing	
A constant value	

18. Summarize at least three concepts that were reinforced during this activity.