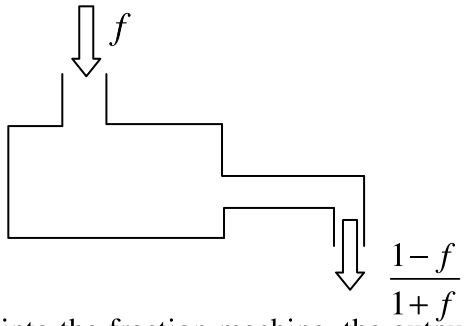
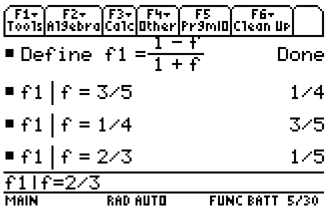
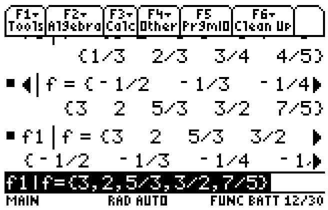
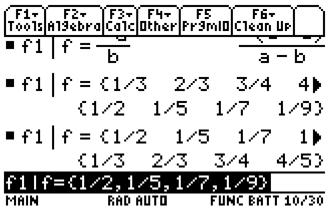
SOLUTIONS TO YEAR 10 PROPLEM SOLVER FRACTION MACHINE:



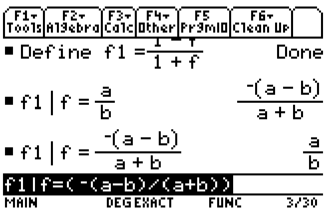
When $\frac{3}{5}$ is fed into the fraction machine, the output is $\frac{1}{4}$, but when this output is fed back into the machine, the original fraction $\frac{3}{5}$ is obtained. In the case of $\frac{2}{3}$, the output is $\frac{1}{5}$, which results in $\frac{2}{3}$ when it is processed. Students could conjecture that an even number of processes will always result in the original fraction. The following screens show how Define and Lists features of a CAS calculator can be used to investigate the fraction machine:

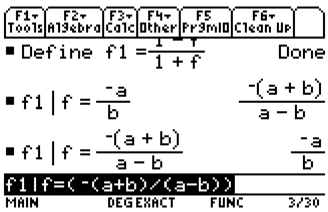






Entering the fraction $\frac{a}{b}$, then processing the output shows that this indeed results in the original fraction $\frac{a}{b}$. Similarly, entering the fraction $-\frac{a}{b}$ results in the same fraction after an even number of processes.





When the input fraction is $-\frac{3}{5}$, the output is 4. Students may conjecture that a positive fraction can never give rise to a whole number because the numerator $-(a-b)$, that is, $b-a$ will always be less than the denominator $a+b$. However, for a negative fraction, a whole number will always result if a and b are odd integers and the difference between a and b is 2. For example, we predict that whole numbers will result for the fractions $-\frac{5}{7}, -\frac{7}{9}, -\frac{9}{11}, \ldots\ldots$

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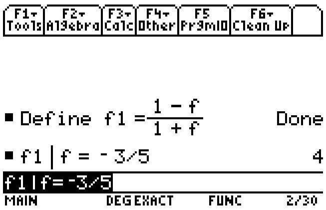

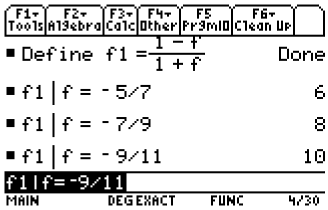

Some students may conjecture that the whole number will result if $b-a=1$ and $a<b$ which is shown in the following screen:
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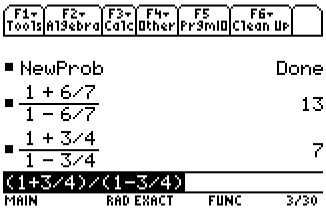

It is another valid generalisation.

The following generalisations can be made after having conducted the fraction machine investigation:

1. When the proper fraction being fed into the machine is in the form $f=\frac{a}{b}$ or $f=-\frac{a}{b}$, where a and b are positive integers, the fraction machine works in a cyclic manner. It will always return the original fraction after the even number of processes and it will return $\frac{b-a}{a+b}$ or $\frac{a+b}{b-a}$ respectively after the odd number of processes.

2. In order to obtain the whole number as an output we need to feed a negative fraction $f=-\frac{a}{b}$. This will return $\frac{a+b}{b-a}$. The following conditions must occur:
   a) $b-a=1$ and $a<b$
   b) a and b are both odd integers and $b-a=2$.