Properties of Logarithms

ID: 9607

Time required
75 minutes

Activity Overview

Students use algebra and graphing to discover the properties of logarithms. Given an equation in a and b such as $b = a^2$ and a table of values for a, students define x = log a and y = log b and graph the resulting sets on the coordinate plane, graphically "taking the log of both sides." Finding the equation and working backwards reveals an equation that expresses one of the properties of logarithms. Seeing a power function transformed into a linear function visually reinforces the effect of the log.

Topic: Exponential & Logarithmic Functions & Equations

- *Derive algebraically the rules for products, quotients, and powers of logarithmic expressions.*
- *Verify graphically the rules for products, quotients, and powers of logarithmic expressions.*

Teacher Preparation and Notes

- *This activity is appropriate for students in Algebra 2 or as a review for Pre-calculus. Prior to beginning this activity, students should be familiar with the definition of a logarithm and have experience with simple exponential equations of the form $b = a^x$, as well as the exponent rules.*
- *This activity requires students to enter formulas, graph scatter plots, draw a line and find its equation, use the Graph Trace tool, and draw points on a Graphs screen. If students are unfamiliar with these basic functionalities of the handheld, extra time should be taken to explain them.*
- ***To download the student and solution TI-Nspire™ documents (.tns files), go to education.ti.com/exchange and enter "9607" in the keyword search box.***

Associated Materials

- *PropertiesOfLogs.tns*
- *PropertiesOfLogs_Soln.tns*
- *PropertiesOfLogs_Student*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

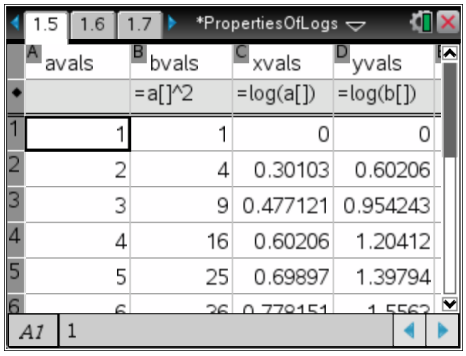
- *Properties of Logarithms (TII-Nspire™ technology) — 16062*
- *Exponentialis Logarithmus (TI-Nspire™ technology) — 9207*
- *What is Log? (TI-Nspire™ technology) — 16055*

Problem 1 – The Power Property of Logarithms

The properties of logarithms are introduced as a parallel to the properties of exponents on page 1.2.

Page 1.3 poses the problem: *How can you simplify the logarithm of a power?* Students should see that there might be some way to separate the exponent, 2, from log(*a*) in this expression.

On page 1.5, students should enter a formula in Column B to calculate the value of $b = a^2$ from the values of *a* in Column A. (They should enter **=a[ ]^2**.) Then, students should graph the data from Columns A and B as a scatter plot on page 1.7. They will see that the data is shaped like a parabola, as they would expect for the function $b = a^2$.



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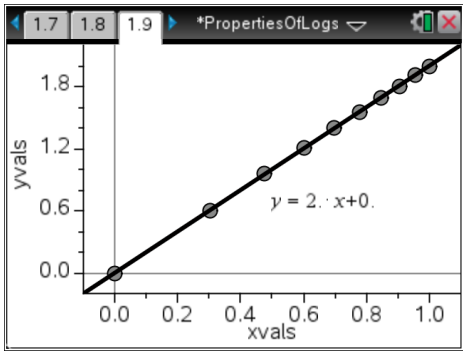
See Note 1 at the end of this lesson.

Page 1.8 directs students to define $x = \log(a)$ and $y = \log(b)$. On page 1.5, they should enter **=log(a[ ])** in the formula bar for Column C and **=log(b[ ])** in the formula bar for Column D. Alternatively, they could enter **=log(avals)** and **=log(bvals)**, respectively.

Students should then plot the values in Columns C and D in a scatter plot on page 1.9



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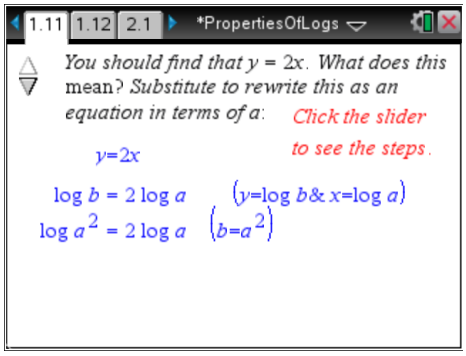
See Note 2 at the end of this lesson.

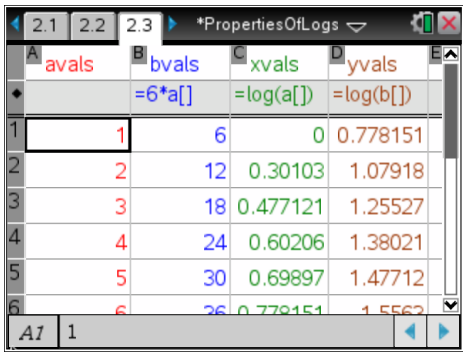
By drawing a line through the points in the plot, students make the surprising discovery that $y = 2x$!

A few simple substitutions, shown on page 1.11 are required to rewrite this equation in terms of *a* only. By first substituting log *b* for *y* and log *a* for *x*, then substituting a^2 for *b*, the *Power Property of Logarithms* is revealed. The property is restated on page 1.12.



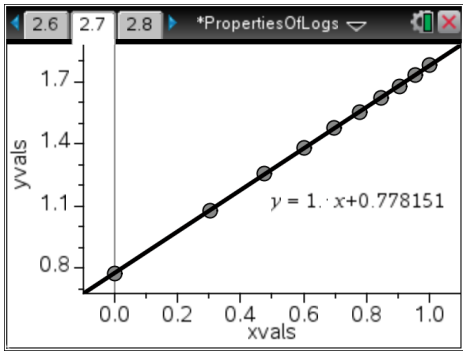
Problem 2 – The Product Property of Logarithms

In this problem, students repeat the process they used in Problem 1 to discover what happens when they take the logarithm of a product. Students should enter either **=6*a[ ]** or **=6*avals** in the formula bar of Column B on page 2.3. The data is linear, with a *y*-intercept of 0 and a slope of 6, as expected.



Students should enter **=log(a[ ])** in the formula bar for Column C on page 2.3 and **=log(b[ ])** in the formula bar for Column D. Alternatively, they could enter **=log(avals)** and **=log(bvals)**, respectively.

Students should then plot the values in Columns C and D as a scatter plot on page 2.7.



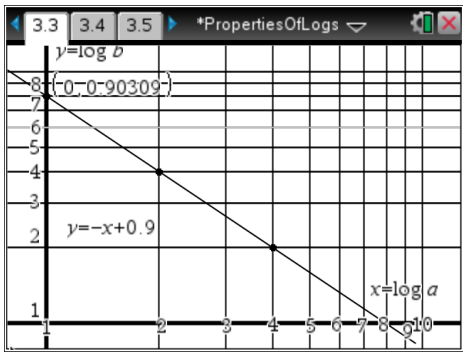
By drawing a line through the plot, students discover that it is a linear function with a slope of 1 and a *y*-intercept of 0.778151.

Page 2.9 directs them to raise 10 to the 0.778151 power. With this hint, students should realize that 0.778151 is log 6.

Armed with this information, students can perform substitution to rewrite the equation $y = \log 6 + x$ in terms of a, revealing the Product Property of Logarithms, which is restated on page 2.11. If they have difficulty with the substitution, students should refer back to page 1.11.

Problem 3 – The Quotient Property of Logarithms

In this problem, students experiment with log-log "paper," a grid with a logarithmic scale on both the *x*- and *y*-axes.



Verify that the value of the line marked 2, for example, is indeed log 2.

Discuss why the axes are marked 1 instead of 0. (Because log 1 = 0).

Explain that on this grid, *x* is already equal to log *a* and *y* is already equal to log *b*, because the lines are placed at the logarithms of the values.

Once students understand the concept of log-log paper, challenge them to graph the function $y = \frac{8}{x}$. Since it is difficult to estimate logarithmic distance in between the lines, they should graph only those points with whole number coordinates, namely (8, 1), (4, 2), (2, 4), and (1, 8).

By drawing a line through the plot, students discover that it is a linear function with a slope of –1 and a *y*-intercept of 0.90309.

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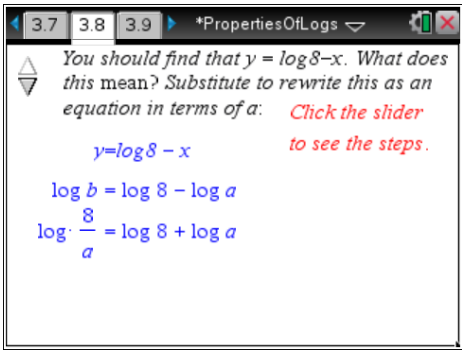
See Note 3 at the end of this lesson.

On page 3.7, students should raise 10 to the 0.90309 power and realize that 0.90309 is log 8.

Armed with this information, students can perform substitution to rewrite the equation $y = \log 8 - x$ in terms of *a*, revealing the Quotient Property of Logarithms, which is restates on page 3.9. If they have difficulty, they should refer back to page 1.11.

Explain to students that these properties apply to logarithms with any base, not just base 10.



TI-Nspire™ Navigator™ Opportunities

Note 1

Question 1, *Live Presenter*

Use Live Presenter to demonstrate how to enter the formulas in the *Lists & Spreadsheets* page.

Note 2

Question 1, *Live Presenter* and *Class Capture*

Use Class Capture to monitor student progress as the scatter plot and linear regression is created. Use Live Presenter to aide in the discussion of this question.

Note 3

Question 3, *Live Presenter* and *Class Capture*

Use Live Presenter to demonstrate how to plot the points on the graph as well as to draw the line passing through the points. Use Class Capture to help aide in the discussion of the questions that follow on the student worksheet.