

About the Lesson

In this activity, students use *Cabri™ Jr.* to explore tessellations of triangles and quadrilaterals. Students will use the rigid transformations of translations, reflections, and rotations to accomplish the task. As a result, students will:

- Use technology to verify that adding the number $-x$ is equivalent to subtracting x .
- Solve one-step linear equations of the form $x + a = b$ where a and b are real numbers.



Vocabulary

- translation
- reflection
- rotation

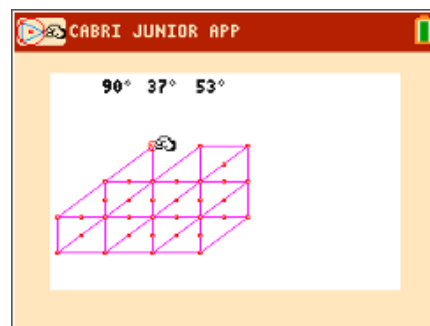
Teacher Preparation and Notes

- A tessellation is a tiling pattern that covers the plane without any gaps or overlaps. Tessellations are formed by performing repeated translations, reflections, and/or rotations on some basic shape (e.g., a triangle). Where the tiling shapes meet (either at vertices or along edges), exactly 360° must be accounted for by the angles of the shapes.
- Measurements in Cabri Jr. can display 0, 1, or 2 decimal digits. If 0 digits are displayed, the value shown will round from the actual value. To change the number of digits displayed:
 1. Move the cursor over the value so it is highlighted.
 2. Press \oplus to display additional decimal digits or \ominus to hide digits.

Activity Materials

- Compatible TI Technologies:
 - TI-84 Plus*
 - TI-84 Plus Silver Edition*
 -  TI-84 Plus C Silver Edition
 -  TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Tessellations_TI84_Student.pdf
- Tessellations_TI84_Student.doc
- RECTNGL.8xv

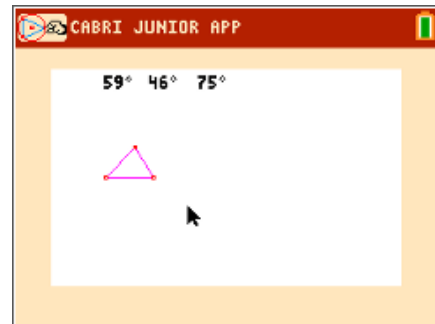
Tech Tip: Before beginning the activity, the *Cabri™ Jr.* file RECTNGL.8xv needs to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE Software.

Problem 1 – Triangles

In the first problem, reflections and translations are repeatedly applied to a scalene triangle, which is the basic geometrical component of the pattern. Note that the reflection of the original triangle forms a quadrilateral element that can be used with translations only to tile the entire space.

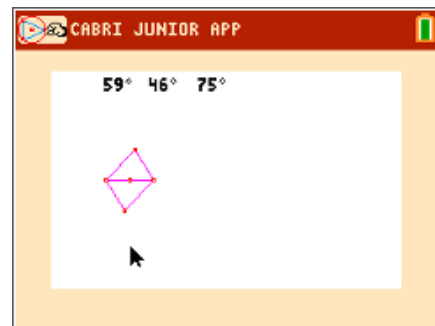
Step 1: Open a new *Cabri Jr.* file. Use the **Triangle** tool to construct a scalene triangle by pressing `[window]` and selecting **Triangle**.

Use the **Angle** tool to measure the angles of the triangle by pressing `[graph]` and selecting **Measure > angle**. Now, select the three vertices that determine the desired angle.

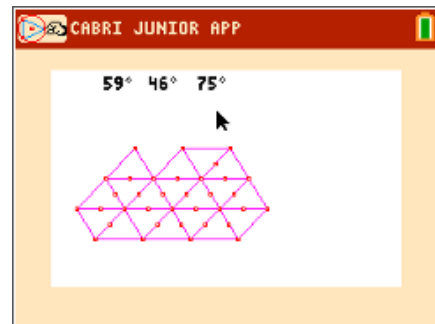


Step 2: Use the **Midpoint** tool to construct the midpoint of one side of the triangle by pressing `[zoom]`, selecting **Midpoint**, then selecting the desired side of the triangle.

The **Symmetry** tool performs a reflection through a point (which is equivalent to a half-turn or 180° rotation). Use this tool to reflect the triangle through the midpoint by pressing `[trace]` and selecting **Symmetry**. Now, select the triangle and then the midpoint.



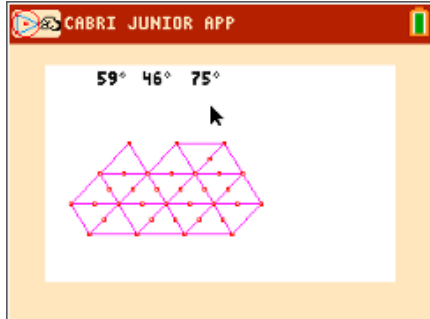
Step 3: Continue to make copies of the triangle using the **Midpoint** and **Symmetry** tools. Cover a portion of the screen with the tessellation.





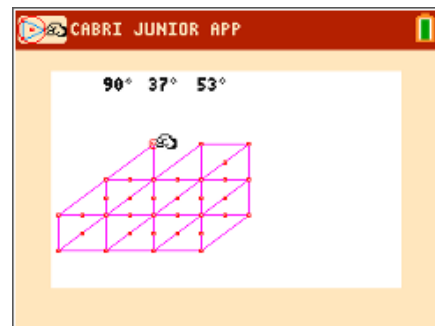
1. Construct a scalene triangle and measure its angles. Find the midpoint of one side. Use the **Symmetry** tool to create a tessellation.

Make a sketch of your triangle tessellation.



Answer:

- Step 4:** Drag a vertex of the original triangle.



2. Drag a vertex of the original triangle. Does it matter what type of triangle (equilateral, isosceles, scalene) you use to create the tessellation?

Answer: No, it does not matter the type of triangle used to create the tessellation.

3. How many angles come together at one vertex of the tessellation?

Answer: 6

4. List the measures of all the angles that come together at one vertex of the tessellation.

Sample Answer: 90° , 30° , 0° , 90° , 30° , 60°

5. Find the sum of the angles at one vertex of the tessellation. Will you get the same result regardless of the shape of the initial triangle? Explain why or why not, using theorems about angles and lines.

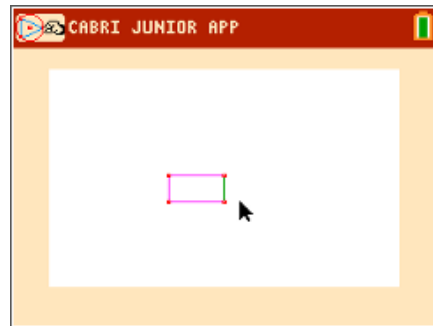
Answer: 360° ; If you label the three angles of the original triangle with the variables, a , b , and c , the first reflected triangle will have the same angles, since reflection and rotation preserves angles. From there, use the vertical-angle and alternating-interior-angle theorems to identify the measure of each angle in the tessellation. In all cases, the sum of the angles of each vertex will be $2(a + b + c)$, which must total 360° because the sum of the interior angles of a triangle is 180° .

Problem 2 – Rectangles

In this problem, rectangles are used as the basic geometrical unit. In the first case, reflections across vertical and horizontal line segments are used to create the tessellation. In the second case, horizontal and vertical translations are used involving integer multiples of the length and width of the original rectangle. The third case also involves the same translations but with alternating rows shifted by a half-length relative to adjacent rows. In the final case, rotations are combined with 90° rotations.

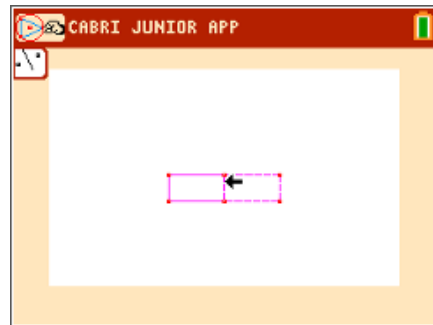
- Step 1:** Open the file *RECTNGL*. Students will be using three different transformation tools to tessellate the rectangle.

After each file is saved with a new name, students may drag a vertex of the rectangle to adjust its dimensions.

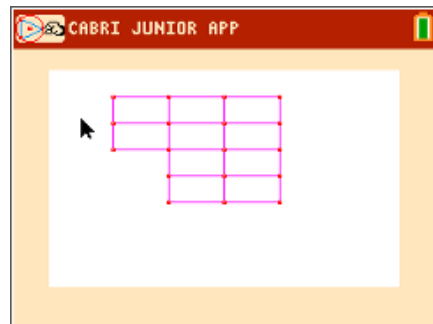


- Step 2:** First, use the **Reflection** tool. Save the file with the name *RECTREFL* so the original file will be available for the next transformation.

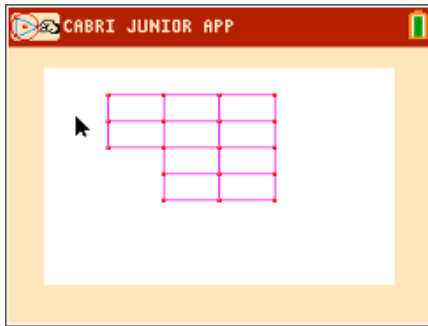
To perform the reflection, select the rectangle and a line of reflection. The line of reflection may be a line, a segment, or a side of the rectangle



- Step 3:** Cover a portion of the screen with copies of the rectangle.



6. Save the file with the name *RECTREFL* and use the **Reflection** tool. Make a sketch of the tessellation.



Answer:

- Step 4:** Now use the **Translation** tool. Open the *RECTNGL* file and save it with the name *RECTTRAN*.

A translation uses a vector, which can be thought of as a “directed” line segment.

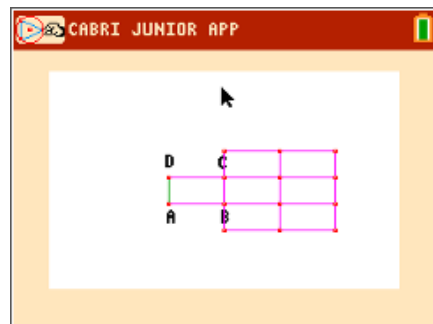
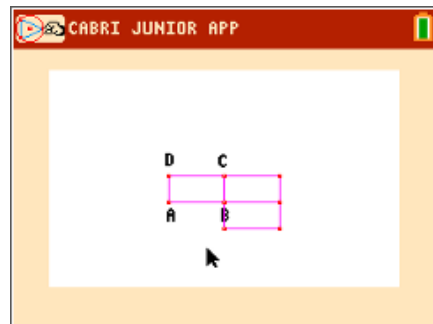
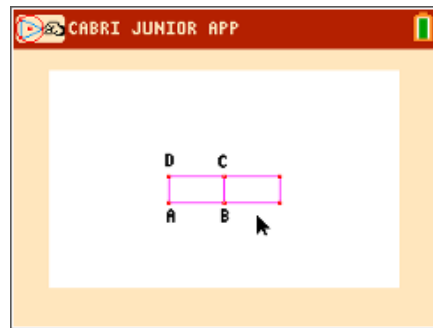
To perform the translation, select the rectangle and the endpoints of a line segment. The order of selection of the endpoints determines the direction of the translation.

Use the endpoints of one side of the rectangle for the translation vector.

In this figure, points *A* and *B* were selected in that order to produce the translation.

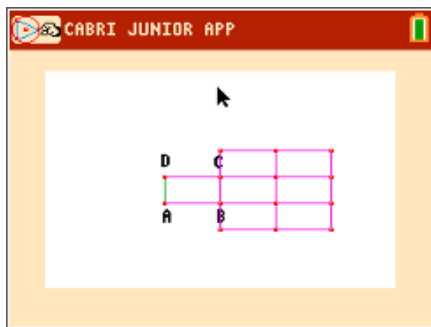
- Step 5:** In this figure, the rectangle on the right side was selected. Then points *C* and *B* were selected in that order to produce the translation.

- Step 6:** Cover a portion of the screen with copies of the rectangle.





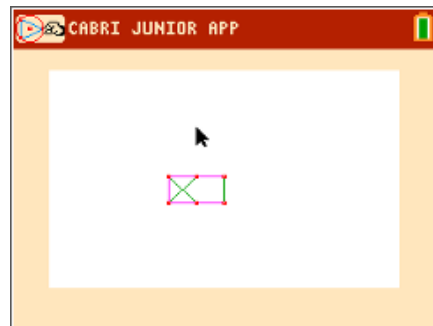
7. Save the *RECTNGL* file with the name *RECTTRAN* and use the **Translation** tool. Make a sketch of the tessellation.



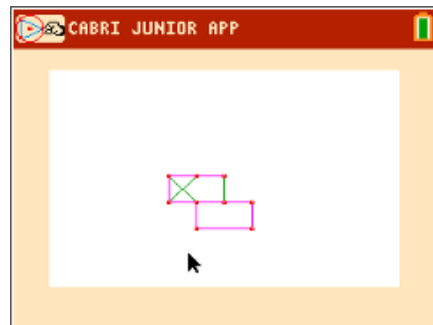
Answer:

Step 7: Next, we will try a different translation. Open the *RECTNGL* file and save it with the name *RECTTRX*.

Construct the midpoint of each long side of the rectangle. Create two diagonal segments as shown.



Step 8: Use the endpoints of one diagonal segment as the translation vector.



8. Save the *RECTNGL* file with the name *RECTTRX*. Construct the midpoint of each long side of the rectangle. Create two diagonal segments and use the **Translation** tool. What pattern will result?

Answer: A tiling that looks like a brick wall.

9. How many angles come together at one vertex of the tessellation for the cases in Questions 6, 7, and 8?

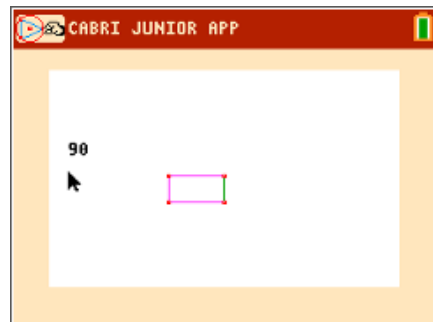
Answer: In the cases for Questions 6 and 7, there are four angles that come together at each vertex. In the case of Question 8, there are three angles that come together.

10. Find the sum of the angles at one vertex of the tessellation for each of those cases.

Answer: The sum is 360° in all three cases. For Questions 6 and 7, the sum is $4(90) = 360$, and in the case of Question 8, the sum is $2(90) + 180 = 360$.

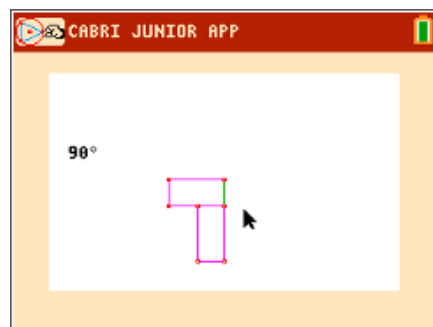
Step 9: Finally, use the **Rotation** tool. Open the *RECTNGL* file and save it with the name *RECTROT*.

Use the **Alpha-Num** tool to place the number 90 on the screen (press `[alpha]` to access numerical characters). Press `[enter]` to start and end the text.

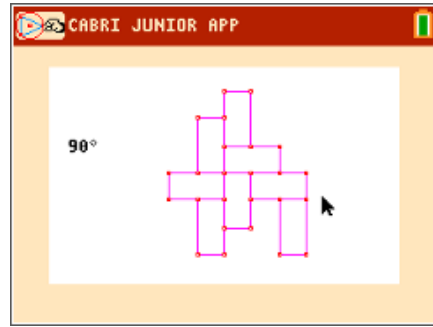


Step 10: To perform the rotation, select the rectangle, the number 90, and a point as the center of rotation. Use a vertex of the rectangle for the center of rotation.

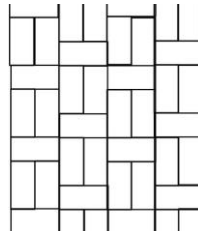
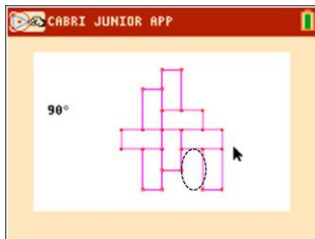
The rotation is completed in a counter-clockwise direction.



Step 11: Cover a portion of the screen with copies of the rectangle.



11. Save the *RECTNGL* file with the name *RECTROT* and put the number 90 on the screen. Use the **Rotation** tool. Try to construct a tessellation. Is it possible to construct a tessellation using rotation that covers the entire plane, regardless of the shape of the initial rectangle? Try to find a tessellation that is a repeating pattern of rectangles of both orientations, assuming the length is twice the width.



Answer:

It is possible, but it is also easy to get stuck when using rotations to form the tessellation, particularly if the long dimension of the original rectangle is not an integral multiple of the narrow dimension. (E.g., in the left figure above, there would be no way to tile the area shown if the length wasn't twice the width.) An example of a tessellation formed by repeating units is shown on the right.

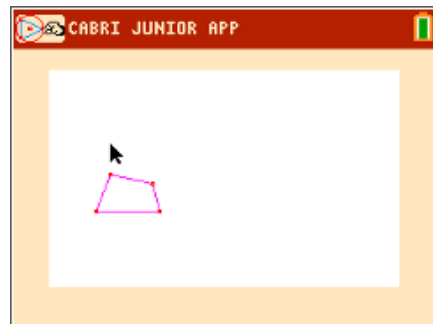
12. What other tools from the Transformation menu could be used instead of the **Symmetry** tool to create the same result?

Answer: reflections

Problem 4 – Quadrilaterals

Step 1: Have students open a new *Cabri Jr.* file.

Students are to construct a quadrilateral using the **Quad.** tool.

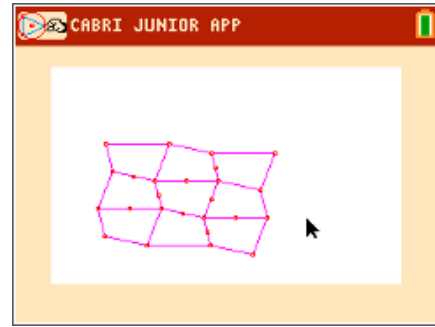




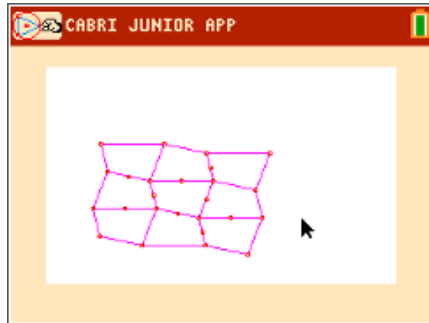
Step 2: Use any of the Transformation tools to create a tessellation with the quadrilateral.

Cover a portion of the screen with copies of the quadrilateral.

Make a sketch and record observations on the worksheet.



13. In a new *Cabri Jr.* file, construct a quadrilateral and create a tessellation using any of the Transformation tools. Make a sketch of the tessellation.



Sample Answer:

14. What Transformation tool(s) did you use to create the tessellation?

Sample Answer: One possible answer is reflection through a point using the Symmetry tool.

15. How many angles come together at one vertex of the tessellation?

Answer: 4

16. What are the measures of these angles? Find the sum of the measures.

Sample Answer: 120° , 120° , 60° , 60° ; 360°

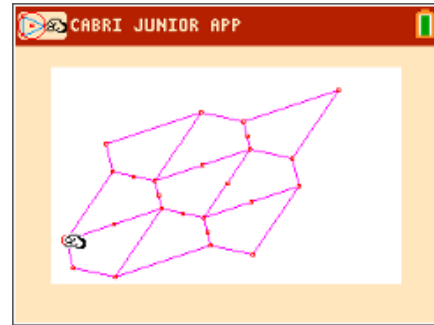
Step 3: How many angles come together at one vertex of the tessellation?

What are the measures of these angles?

Find the sum of the measures.



Step 4: Drag a vertex of the original quadrilateral and observe the results.



17. Drag a vertex of the original quadrilateral and observe the results. Complete the conjecture:

The sum of the angles that come together at one vertex of a tessellation is _____. Justify your conjecture, using theorems about angles and lines.

Answer: 360° ; If you label the four angles of the original quadrilateral with variables, the first reflected quadrilateral will have the same angles, because reflection and rotation preserves angles. From there, use the vertical-angle and alternating-interior-angle theorems to identify the measure of each angle in the tessellation. In all cases, the sum of the angles of each vertex will be the sum of the interior angles of the initial quadrilateral, which must total 360° .