## Reverse Parking Rover

## Teacher Notes \& Answers

$$
\begin{array}{lllll}
7 & 8 & 9 & 10 & 11
\end{array}
$$



## Introduction

There are already many commercially available vehicles that will reverse (parallel) park autonomously. In this activity your aim is to get Rover to reverse park into a pre-determined parking space. To achieve this reverse park you will need to get Rover to perform one partial left curve and one partial right curve whilst reversing. The size of each curve can be determined by a combination of vehicle and parking locations. Rover must park within the given space and without hitting any obstacles.

## Teacher Notes:

## Equipment

- Roll of Kraft or butcher's paper at least 60 cm wide. Students (groups) will need at least a 3 m length to record their results. Students should be encouraged to use both sides of the paper but should also include at least one sample measurement with their completed task.
- TI-Innovator Rover with TI-Innovator
- TI-Nspire calculator.
- Erasable marker (Expo or similar) for Rover (Must fit neatly inside Rover's pen holder)
- 1 m ruler +30 cm ruler
- String (optional) to measure the curved distance that Rover travels. An alternative is to trigonometry to calculate the curved distance or use both to compare 'accuracy'.


## Tips

Students should be made aware of the final challenge and the importance of accuracy throughout the task. Check all measurements, calculations and the reproducibility of results including what happens when trying to turn in the opposite direction. Students will need to decide appropriate tolerances when doing the final reverse parking test.

The TI-Innovator Rover has a number of standard "Drive" commands; in this activity some speciality commands will be used. The "SET RV.MOTORS LEFT \# RIGHT \#" command provides control over the power delivered to each motor. Differentiating the power delivered to each motor means Rover will drive in a curve, a curve that can be closely approximated to a circle. Alternating this power distribution at the right times will make Rover drive in an ' $S$ ' type configuration required to achieve a successful reverse park.

A number of tasks must be completed in order to build an appropriate theoretical model for practical testing and revision.

## Task 1:

Identify a relationship between the power delivered to Rover's wheels and the length and curvature of the arc.

## Task 2:

Plan your reverse parking strategy, implement the theoretical models, evaluate and review practical results.
Task 3:
Driving Test - Reverse Park Rover!

## Task 1: Power to the Wheels

Open the TI-Nspire document: Reverse Park.
On page 1.1 use the menu to insert a new program
[Menu] > Functions \& Programs > Program Editor > New
Call the program: Reverse
The first line of any Rover program starts with:

## Send "RV CONNECT"

The next step is to distribute the power sent to each of Rover's motors.

> Hub > Rover (RV) > RV Control > Send "SET RV.MOTORS"

The power delivered to each motor is determined by a single 'byte' of information $(-255$ to +255$)$. Negative numbers rotate motors counterclockwise (CCW), positive numbers rotate motors clockwise (CW).

The motors are identified as LEFT and RIGHT:

$$
\text { Hub > Rover }(\text { RV })>\text { RV Settings > LEFT }
$$

| 奛1: Actions | RADSTIX |
| :---: | :---: |
| d 2: Check 1: Send "SET |  |
| : $=3$ : Define 2 2: Send "READ |  |
| 1: Drive RV |  |
| 2: Read RV Sensors |  |
| 3: RV Settings |  |
| 4: Read RV Path |  |
| 5: RV Color |  |
| 6: RV Setup | T-Output |
| 7: RV Control | T-Input |
| 8: Send "CONNECT RV" |  |
| 9: Send "DISCONNECT RV" |  |



The completed command should appear as follows:

## Send "SET RV.MOTORS LEFT 100 RIGHT -255"

Hold Rover up and look at the right hand side (Switch side). Which way will the right hand wheel drive Rover when it is rotating clockwise? Do the same for the left hand side of Rover. Make a note of values that need to be sent to Rover's motors to drive forward / backward.

This 'drive' command will continue to execute until a STOP command is generated. For this first example the command will be executed for a period of 2.5 seconds. A wait command is therefore required:
Hub > Wait

Followed by a STOP command:
Hub > Rover (RV) > Drive RV > Send "RV STOP"


Place Rover on a smooth, level, hard surface and press Ctrl + R to save and run the "Reverse" program. Make sure Rover has space to move backwards in an arc.

Question: 1
Suppose the power to the left wheel is increased, what would happen to the curvature of the arc?
Explain your answer.
Answer: If the power to the left wheel is increased then the radius of the arc will increase. Increasing the power means the motor rotates more frequently in the same amount of time. The number of rotations translates to the distance the wheel will propel Rover. If the left wheel receives less power it will drive the left-hand side of Rover a shorter distance than the right resulting in a curve, equal power will drive Rover in a straight line.

Teacher Notes: Students should choose their language carefully when responding to this question. To simply say that "Rover draws a bigger arc" is somewhat ambiguous. "Bigger Arc", does the arc come from a circle of larger radius (different curvature) or is the arc simply longer, perhaps both.
If students extend the wait time significantly they will find that Rover does not draw a 'perfect' circle. This is due to a number of factors, in part due to slippage (traction) of the wheel. This imperfection is natural and has very little impact on student results as they approximate the arc as part of a circle.

The next stage of this investigation is to establish a relationship between the distribution of power to Rover's wheels, the size of the circle's radius and the distance that Rover travels. You will need to ensure your measurements are as accurate as possible and that your calculations are correct. Throughout the investigation you should consider the reproducibility of your results, the amount of slip, 'road' surface, direction, relative charge on Rover's batteries and other factors that naturally produce variability in real events.

Set up a length of paper on a flat, level, hard surface and tape the paper to the surface to reduce the amount of movement generated in the paper. Set Rover up in an appropriate position so that the path to be traced out will be completely contained on the paper. Try doing a couple of test runs before inserting the marker into Rover's pen holder.

When you are ready, place the marker into Rover's pen holder and complete the first trial run using the following:
Left Power $=100$
Right Power $=-255$
Wait Time $=2.5$
Use geometric techniques to measure the radius of the arc and physical and theoretical techniques to measure the length of the arc. The TNS file contains a series of diagrams to help understand and measure the radius.

RADIUS: Place three points on the arc, as spread out as possible. Draw lines connecting the three points to form a triangle. Draw perpendicular bisectors to each of these lines (sides). The perpendicular bisectors should intersect at one point, (approximately) the centre of the circle.

Tip 2
ARC: To measure the length of the arc traced out by Rover use a piece of string or flexible chord laid out carefully along the arc then measure this length using a ruler. Compare this result to that obtained using trigonometry.

Teacher Notes: A quick review of how the circumcentre of a triangle is created and why it works is included in the TI-Nspire file. Placing three points on the arc is the reverse construction of the circumcentre of a triangle. Students can use the 'step' slider to progressively reveal the construction. When completed accurately the perpendicular bisectors should all converge on a region about the size of a 5 to 10 cent piece. Students can then determine the approximate radius of the arc by checking the distance to each of the three points.


## Question: 2

Record your results from your each test run in a table or spreadsheet. Vary the power delivered to the left wheel in incremental amounts and determine the radius and arc length for each. [Include samples of your work]

| Test | Left | Right | Radius | Distance |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | 60 | -255 | 11.7 | 28.7 |
| $\mathbf{2}$ | 70 | -255 | 13.2 | 30.4 |
| $\mathbf{3}$ | 80 | -255 | 14.6 | 31.4 |
| $\mathbf{4}$ | 90 | -255 | 16.7 | 32.6 |
| $\mathbf{5}$ | 100 | -255 | 18.5 | 33.7 |

## Teacher Notes:

The answers provided here should be used as a guide. Each Rover is unique and therefore will produce slightly different results. This provides a significant advantage in the educational space where teachers can use this to ensure that students are doing 'their own work'.
It should also be noted that a 'Wait' time of 2.5 seconds means that each arc is less than a semi-circle. If the wait time is increased to 3 seconds the arc can become slightly larger than a semi-circle, this may cause confusion for some students if they are trying to determine the arc angle using trigonometry.
The next step is to determine a relationship between the power delivered to the left wheel and the corresponding size of arc radius.

## Question: 3

Use the Data and Statistics application to graph the relationship between the power delivered to the left wheel and the corresponding radii.
a) Use an appropriate regression model to determine the relationship and the corresponding correlation coefficient.

Answer \& Teacher Notes:
Answers will vary depending on data collected. Students may choose to apply linear regression. The sample of data produced in this series of trials generated an $R^{2}$ value of 0.977 , a very strong positive correlation. A residual plot however may highlight some necessary reflection on the model chosen. Students should consider the practical situation, if the power to the left wheel is further increased it will eventually equal that delivered to the right wheel resulting in a straight line. A linear model simply predicts a larger radius regardless of the increase in power.


As students identify the power required for the reverse park the chosen model can become critical. If the eventual power selected falls within the existing measurements (interpolation), the linear model will suffice, however if the power required is outside those tested (extrapolation), students should be prepared for significant error and reflect on their chose model. At the very least students should discuss the limitations of the model they have selected.
b) Include a graph of the data, regression equation and corresponding residual plot. Discuss the result including reference to the accuracy of the data and appropriateness of the mathematical model.

Answer: Included above.

## Question: 4

Use your data to determine Rover's approximate speed for each arc and the corresponding relationship between the power delivered to Rover's wheels and the overall speed. This relationship can be used to help plan your reverse park strategy.

Answer: Answers will vary depending on student data. Students may use the arc length directly in their calculations since all the wait times are the same. ( 2.5 seconds)

| Power | 60 | 70 | 80 | 90 | 100 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Arc Length | 28.7 | 30.4 | 31.4 | 32.6 | 33.7 |
| Speed | $11.5 \mathrm{~cm} / \mathrm{s}$ | $12.2 \mathrm{~cm} / \mathrm{s}$ | $12.6 \mathrm{~cm} / \mathrm{s}$ | $13.0 \mathrm{~cm} / \mathrm{s}$ | $13.5 \mathrm{~cm} / \mathrm{s}$ |

## Task 2: Reverse Park Strategy

It's time to start planning your reverse park. By alternating the power distribution to Rover's wheels at the right time it is possible to get Rover to drive along an S type curve as shown below. The dimensions of the parking space are provided as $45 \mathrm{~cm} \times 19 \mathrm{~cm}$; the gap between the two parked vehicles is 55 cm . You can place Rover anywhere outside the parking space in preparation for its reverse park, further forward, further backward or even further from the kerb. Your goal is to have Rover self-reverse park within the confines of the parking space as close to the kerb as possible and preferably equally spaced between the two parked vehicles.


The TI-Nspire file contains a dynamic model of the situation
Car A: Move the point near the rear left corner to change separation.
Car B: Move the point near the front left corner to change separation.
Point C : Move to change the radius for the reverse park.
Point R: Change the distance between Rover and kerb / car.
Slider A: Change the angle to increase/decrease arc length.


Target: The light grey rectangle is the initial target space for Rover.

After experimenting with the interactive diagram, determine an approximate radius and arc angle.

## Question: 5

Construct a detailed diagram including approximate measurements from the interactive diagram (radius \& arc angle) to show that Rover may reverse into the parking space.
Note: Rover is not a 'point'; it is a vehicle with length and width. The diagram included here would not produce a successful park as Rover would hit the rear corner of Car A!

Answers will vary. Students should however include the following considerations:

- Rover has a reasonable clearance around Car A (As referenced in the note to students.)
- Rover will not collide with Car B
- Rover is finishes as close as possible to the gutter

Teacher Notes: The degree to which trigonometry is used to complete this problem versus scaled diagrams is up to the teacher / student. The TI-nspire diagram is to be used as a guide only. TI-Rover boxes can be used for Car A and Car B .

Additional lines now need to be added to the code that drives Rover.
Return to the Reverse program and repeat the SET RV.MOTORS and Wait commands accordingly. It is important to check that swapping the power distribution produces the same results.
(Values shown opposite are for illustrative purposes only.)
Use the values determined in your experimentation and corresponding calculations to set up the reverse park.

| 1.1 |  | P |  |
| :---: | :---: | :---: | :---: |
| ${ }^{*}$ reverse 5 |  |  |  |
| ```Define reverse()= Prgm Send "CONNECT RV" Send "SET RV.MOTORS LEFT 100 RIGHT -2 Wait 2.5 Send "SET RV.MOTORS LEFT 250 RIGHT -1 Wait 2.5 Send "RV STOP" EndPrgm``` |  |  |  |

## Question: 6

Detail your calculations and corresponding power and wait time settings.
Answers: (Sample)
Required Radius: 30 cm Reg.Eqn: $30=0.191 \times$ Pwr -1.14 ... Power $\approx 160$ (Extrapolation*)
Angle Required: $45^{\circ} \quad$ Calculated Dist: $23.4 \mathrm{~cm} \quad$ Calculated Speed: $16.4 \mathrm{~cm} / \mathrm{s} \quad$ Calculated Time: 1.4 s
*Extrapolation - This will test the accuracy and limitations of the linear model!

Complete a couple of practice runs with your settings and fine tune them accordingly.

## Question: 7

Insert a felt tip marker in Rover's pen holder to record the reverse path and check the measurements accordingly.
Answer: Students submit the path that Rover followed and reflect on its accuracy compared with their calculations and review power and timing accordingly. Students should be encouraged to submit more than one test run to show how they have further modified their final input values.

Once you are satisfied with the first stage of Rover's reverse park, it is possible to have Rover automatically complete the park by adjusting its position based on the distance to the vehicle in front.

The ultrasonic sensor on the front of Rover can be accessed as follows:

Hub > Rover (RV) > Read RV Sensors > RV.RANGER
Once this command has been called, the measured distance can be retrieved and stored in the calculator.
Hub > Get

The distance can be stored in a variable (example: d ). The distance returned will be measured in metres. Use the SET RV.MOTORS command to drive Rover 'straight' forward for a measured time based on the values set for the LEFT and RIGHT motors and the value stored in $d$.

|  | RAD |
| :---: | :---: |
| * reverse | 11/11 |
| Wait 2.5 |  |
| Send "SET RV.Motors left 250 RIGHT -1 |  |
|  |  |
| Send "RV STOP" |  |
| Send "READ RV.RANGER" |  |
|  |  |
| Send "SET RV.MOTORS LEFT -120 RIGHT 1 |  |
| Wait \# |  |
| Send "RV STOP"\| |  |
| EndPrgm |  |

## Question: 8

Detail your calculations and corresponding power and wait time settings to drive forward an amount stored in the variable ' $d$ '.

Answer: Students may use a variety of techniques to complete this section of the task. They may determine Rover's speed for the corresponding power settings followed by a distance calculation so that Rover is located centrally in the parking space.

Example: Overall Distance $=55 \mathrm{~cm}$. Rover length $\approx 30 \mathrm{~cm}$. Centred distance between Rover, Car A and B: 12.5 cm
Distance to travel forward $=\mathrm{d}-0.125 \mathrm{~m}$. Students will need to be careful if they use the 'Forward' command as this command uses distances measured in decimetres.

## Task 3: Driving Test

Once you are confident that your calculations, measurements and road testing are complete, you're ready to reverse park Rover. Your teacher will provide parking space for your Rover. Set Rover in its place and run your program to see how well you have done!

Teacher Notes: Ideally students will park Rover centrally within the confines of the designated parking space. An applicable "Rover Driving Certificate" is a good reward for this aspect of the task; however just like real driving tests, being able to drive a car doesn't make one a good driver. Similarly, there may be unsuccessful parking situations for students that have performed all the necessary calculations. It is left to the individual teacher to determine the appropriate marking criteria.

