Comparing Linear and Exponential Data

Math Objectives

- Students will use a table and a graph to compare the changes in linear and exponential relationships as *x* increases.
- Students will recognize that as *x* increases, a linear function increases at a constant rate (additively) while an exponential function increases multiplicatively.

Vocabulary

- linear function
- exponential function

About the Lesson

- This lesson involves comparing data from two different scenarios—linear and exponential growth.
- As a result, students will:
 - Complete a table involving prize money scenarios.
 - Examine the table for trends in the growth of each prize money scenario.
 - Examine the graphs to compare the data.

TI-Nspire™ Navigator™ System

- Use Class Capture to compare and contrast linear and exponential functions.
- Use Quick Polls to assess students' understanding throughout the lesson.
- Use Live Presenter to discuss examples as a class.
- Use the Teacher Software to review student documents.

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Comparing Linear and Exponential Data

In this activity, you will use the Lists & Spreadsheet application along with the Data & Statistics application to compare a linear and an exponential relationship.

TI-Nspire[™] Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages

Tech Tips:

 Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Files:

Student Activity Comparing_Linear_and_ Exponential_Data_Student.pdf Comparing_Linear_and_ Exponential_Data_Student.doc

TI-Nspire document Comparing_Linear_and_ Exponential_Data.tns Comparing Linear and Exponential Data

Discussion Points and Possible Answers

Move to page 1.2.

The data in this activity will be generated by examining two scenarios: a prize award that begins with \$10 and increases by \$5 each week for 16 weeks, and a prize award that begins with \$0.01 and doubles each week for 16 weeks.

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The data will be generated by examining two scenarios.

• a prize award which begins with \$10 and increases by \$5 each week for 16 weeks

• a prize award which begins with \$0.01 and doubles each week for 16 weeks

TI-Nspire Navigator Opportunity: *Class Capture and/or Live Presenter* See Note 1 at the end of this lesson.

Teacher Tip: Another possibility for this lesson is to start off with the student activity sheet and have students go through questions 1–6 before using the TI-Nspire document. Students then can proceed to Page 1.3 to finish the lesson. If you want students to generate the data for the two scenarios, see the Tech Tip at the end of this lesson.

TI-Nspire Navigator Opportunity: *Quick Poll* See Note 2 at the end of this lesson.

Teacher Tip: As students are progressing through the table, encourage them to think about the formulas for the amount of the prize award at any week.

 Complete the table for the first five weeks where the first column (Weeks) represents the week number of the prize award. The second column (Adding) represents the amount of the prize award each week from the first scenario. The third column (Doubling) represents the amount of the prize award each week generated by the second scenario. Feel free to use the Scratchpad if necessary.



Weeks	Adding	Doubling
0	10	\$0.01
1	\$10 + \$5 = \$15 \$0.01 * 2 = \$0.02	
2	\$10 + \$5 * 2 = \$20	$0.01 * 2^2 = 0.04$
3	\$10 + \$5 * 3 = \$25	\$0.01 * 2 ³ = \$0.08
4	\$10 + \$5 * 4 = \$30	\$0.01 * 2 ⁴ = \$0.16
5	\$10 + \$5 * 5 = \$35	\$0.01 * 2 ⁵ = \$0.32

Sample Answers: Student work will vary.

TI-Nspire Navigator Opportunity: *Live Presenter* See Note 3 at the end of this lesson.

Teacher Tip: Students might just want to find the value of each table entry instead using a formula. However, if students think of the calculations using a formula dependent on the number of weeks, they will be able to answer the questions that follow more easily.

2. Describe any patterns that you see in the Adding column of the table.

Sample Answers: Answers will vary, but should include: getting larger, increasing by \$5 each time, a linear growth. Some students might find the formula to be 10 + 5 * weeks.

3. Describe any patterns that you see in the Doubling column of the table.

Sample Answers: Answers will vary, but should include: getting larger but very slowly, an exponential growth. Some students might find the formula to be 0.01 * 2^{weeks}.

4. If only looking at the first five weeks, which prize would you rather have?

Answer: If only looking at the first five weeks, I would rather have the adding scenario.

Teacher Tip: As students are progressing through the table, encourage them to think about the formulas for the amount of the prize award at any week.



- 5. Bella says she thinks that after computing more weeks, the better prize to take is the prize that begins with \$0.01 and doubles each week.
 - a. Do you agree?

Answer: Yes, I agree with Bella.

b. How could she justify this?

Sample Answers: The data in the Doubling column, which grow exponentially, will eventually get greater more quickly than the data in the Adding column, especially after it gets to dollars. She might use formulas to justify it.

c. Approximately when might this happen?

<u>Answer:</u> The amount of money in the Doubling column is greater than the amount of money in the Adding column at week 13; but at week 14, the amount of money in the Doubling column jumps way ahead.

Move to page 1.3.

6. The columns are populated with data from each prize scenario. Scroll down the columns and determine if Bella was correct. At what week does the doubling prize become the greater prize?

<u>Answer:</u> At week 13, the Adding column value is 10 + 5 * 13, which equals \$75. The Doubling column value is $0.01 * 2^{13}$, which equals \$81.92.

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- 7. What if the prize was changed to 20 weeks?
 - a. How much prize money would there be for the *adding* scenario? How did you find the amount?

Answer: \$10 + \$5 * 20 = \$110

b. How much prize money would there be for the *doubling* scenario? Explain how you found the amount.

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Answer: $0.01 * 2<sup>20</sup> = $10,485.76
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Move to page 1.4.

MATH NSPIRED

On this page, there are places for two graphs. Move to the bottom of the left side of the page and click to choose the variable "weeks". Move to the left center and click to choose the variable "adding". Move to the right side of the page and create a scatter plot with "weeks" as the horizontal variable and "doubling" as the vertical variable.

8. Describe any similarities or differences in the trend of the data as the weeks increase.

Sample Answers: Answers will vary, but again should include linear growth versus exponential growth. Both graphs go up from left to right.

Tech Tip: You might prefer to have students generate the data in a Lists & Spreadsheet page using formulas. See the steps below.

Step 1:

Select from > New Document >Add Lists & Spreadsheet.

Step 2:

In column A, press \blacktriangle on the Touchpad to move to the cell at the top of column A. Be sure to move to the top of the column. Alternatively, click in the cell at the top of column A.

Step 3:

Type the list name weeks next to the letter A, and press enter.

Step 4:

Move to the cell at the top of column B, enter the list name adding, and press enter.

Step 5:

Move to the cell at the top of column C, enter the list name **doubling**, and press enter.

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Comparing Linear and Exponential Data MATH NSPIRED

Step 6:

Enter data, as shown, into the weeks list.

Place the cursor in cell A1 for column A. Enter the first number. Continue until the number of weeks is 16.

Alternatively, enter the first two numbers in column A. With the cursor in cell A1, hold $\textcircled{\text{shift}}$ and press \checkmark to highlight cells A1 and A2. Press **menu** > **Data** > **Fill**. Then press \checkmark until the cursor is in cell A17. Press **enter** to fill in the data.

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Step 7:

Data for the **adding** and **doubling** lists can be entered recursively using a formula.

For the **adding** column, enter 10 in cell B1. In cell B2, type **■ B 1 + 5** and press <u>enter</u>. Move the cursor back to cell B2. Press <u>menu</u> > Data > Fill. Then press **▼** until the cursor is in cell B17. Press <u>enter</u> to fill in the data.

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13	12	70	40.96	
14	13	75	81.92	
15	14	80	163.84	
16	15	85	327.68	
17	16	90	655.36	
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Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

• How linear and exponential data change and how they compare to each other when the independent variable is the same.



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Note 1

Question 1, *Class Capture and/or Live Presenter*: Use Class Capture to determine whether or not students are experiencing difficulty using the TI-Nspire document. Use Live Presenter to demonstrate the correct procedure for using the document.

Note 2

Question 1, *Quick Poll*: You may wish to send a Quick Poll to see if most of the students are obtaining the correct answers in the table. This will enable you to either stop and clear up any misunderstandings or continue with the lesson.

Note 3

Question 1, *Live Presenter*: You might want to use Live Presenter to show or have a student show how to use the Scratchpad.