Determining Area

ID: 8745

Time required
45 minutes

Activity Overview

Students begins by finding the area of a triangle drawn in the Cartesian plane, given the coordinates of its vertices. Given a set of vertices, students construct and calculate the area of several triangles and check their answers with the ***Area*** *tool. Then they divide a shape into triangles to find the area of a polygon with more sides. Through that process, they explore the properties of the determinant.*

Topic: Matrices

- *Calculate the determinant of a matrix.*

Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 classroom.*
- *Prior to beginning this activity, students should have an introduction to the determinant of a matrix and some practice calculating the determinant.*
- *Information for an optional extension is provided at the end of this activity in the student TI-Nspire document. Should you not wish students to complete the extension, you may delete the extension from the student TI-Nspire document.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8745" in the quick search box.***

Associated Materials

- *DeterminingArea_Student.doc*
- *DeterminingArea.tns*
- *DeterminingArea_Soln.tns*

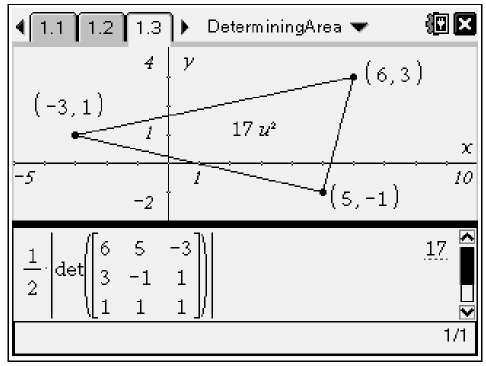
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Cramer's Rule (TI-Nspire technology) — 8793*
- *Triangle in the Matrix (TI-Nspire technology) — 11400*
- *Operating on Matrices (TI-Nspire technology) — 11357*

Problem 1 – Determining the area of a triangle

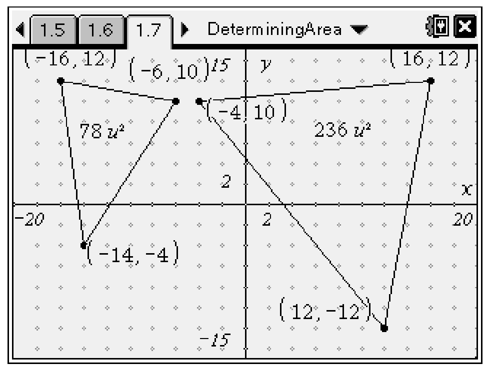
In this problem, students are given a formula for the area of a triangle from the coordinates of its vertices:



$$\text{Area} = \frac{1}{2}\left|\det\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}\right|$$

They apply this formula to find the area of several triangles. They should use the **Coordinates and Equations** tool (**MENU > Actions > Coordinates and Equations**) to find the coordinates of each vertex and complete their calculations in the *Calculator* application at the bottom of page 1.3.

Students then find the area of two triangles with a given set of vertices. Next, they are prompted to check their answers by constructing each triangle and measuring its area using the **Area** tool (**MENU > Measurement > Area**).

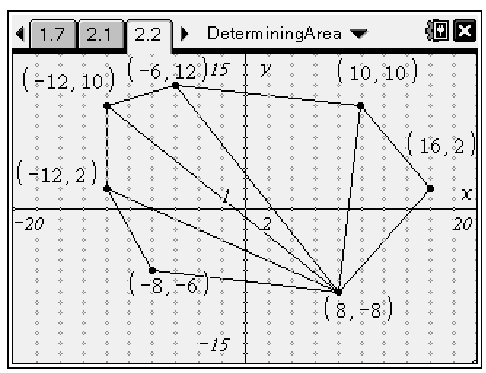


Student Solutions

- Page 1.3: Area = 17 square units
- Page 1.4 Triangle 1: Area = 236 square units
- Page 1.4 Triangle 2: Area = 78 square units

Problem 2 – Area of any convex polygon

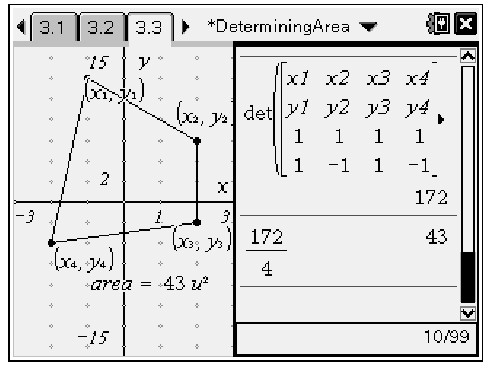
In Problem 2, students will divide a convex heptagon into triangles and use the formula for the area of a triangle to find its area. (This process of dividing an *n*-sided polygon into triangles is part of the proof by induction of a similar area formula for any convex polygon.) Students should use the **Triangle** and **Coordinates and Equations** tools to draw the triangles and find the coordinates of the vertices, respectively. Many different divisions are possible. Page 2.3 is a blank calculator page where they can perform the area calculations.



Student Solution

Area of the heptagon = 422 square units

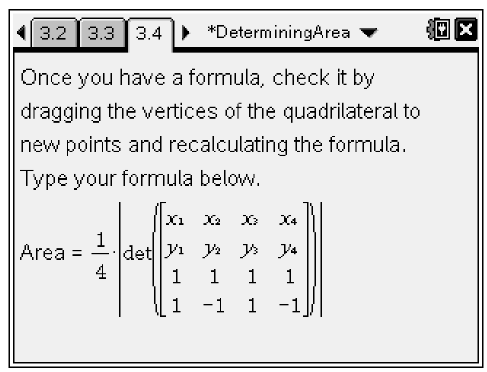
Problem 3 – A formula for the area of a quadrilateral



Pages 3.1 and 3.2 discuss a similar formula for the area of a convex quadrilateral. On page 3.3, students use an example of a quadrilateral with variable vertices to find the formula. The values of the *x*- and *y*-coordinates quadrilateral's vertices have already been stored as ***x*1**, ***x*2**, ***x*3**, ***x*4**, ***y*1**, ***y*2**, ***y*3**, and ***y*4**. These values will update as students move the vertices of the quadrilateral. Students should be able to write the first two rows of the 4×4 matrix with little or no problem.

A common next move is to fill the third and fourth rows with 1s. Explain to students that the determinant of any matrix with two identical rows in 0. In fact, if a row or column is a constant multiple of any other row or column, the determinant of the matrix is 0. If students have experience with using matrices to solve systems of equations, you can tie this fact about determinants back to dependent systems.

Students should try several different combinations of 1 and –1 in the fourth row. The goal is not just a nonzero determinant, but a determinant that is divisible by the area of the quadrilateral. (There are two correct possibilities: both alternate 1 and –1 in the fourth row.)



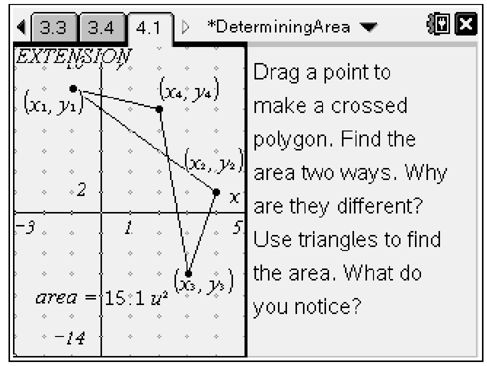
Once a correct combination is found, students should add absolute value bars and a fractional constant to complete their formula. On page 3.4, they are prompted to move the vertices of the quadrilateral and recalculate the area using the formula as a check.

Student Solutions – Page 3.4

$$\text{Area of a quadrilateral} = \left|\frac{1}{4}\det\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix}\right|$$

$$\text{OR } \left|\frac{1}{4}\det\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\right|$$

Extension – Signed area



As an extension, students are given a convex quadrilateral and asked to drag one vertex across the figure so that the quadrilateral is no longer convex. Explain that such a figure is called a **crossed polygon**. Have students find the area of the crossed quadrilateral with both the determinant formula and the **Area** function. Discuss why the results differ. The determinant formula finds the **signed area**, meaning that if part of the polygon crosses over itself, that portion has a negative area. You can illustrate this: cut a polygon cut from a piece of paper two different colored sides. Fold over one corner to show the region with negative area. Challenge students to drag the vertices to create a polygon with a negative signed area.