Math Objectives

- Students will identify the conditions necessary to be able to multiply two matrices.
- Students will follow the example of the process of matrix multiplication.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

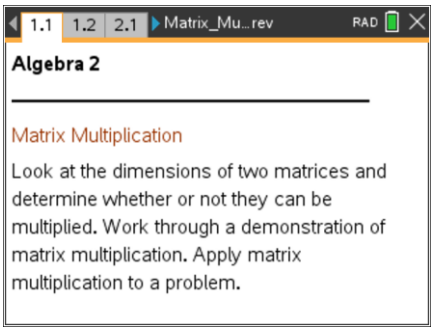
- matrix
- dimensions

About the Lesson

- This lesson involves examining matrix multiplication.
- As a result, students will:
  - Determine whether or not two matrices can be multiplied by examining their dimensions.
  - Follow an example of matrix multiplication to observe the process.
  - Make and justify predictions for finding the elements of an answer matrix.
  - Apply matrix multiplication to a real-world problem.

TI-Nspire™ Navigator™ System

- Use Screen Capture to examine examples of matrix multiplication.
- Use Teacher Edition computer software or Live Presenter to review student documents and discuss examples as a class.
- Use Quick Poll questions to adjust the pace of the lesson according to student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages and panes

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:
Student Activity
Matrix_Multiplication_Student.pdf
Matrix_Multiplication_Student.doc

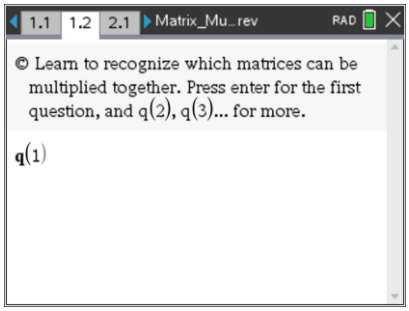
TI-Nspire document
Matrix_Multiplication.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: *Screen Capture* or *Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



1. Press enter to look at question 1. Given the two matrices shown, can they be multiplied? If you think the answer is yes, then type **q(yes);** if you think the answer is no, then type **q(no).** The program will tell you whether or not your answer is correct and why. Repeat the process with new examples by typing **q(2), q(3),** etc., until you are confident that you can tell when matrices can be multiplied.

> **Teacher Tip:** If answer is not readable, have students arrow up two times and then to the left one time to see the entire explanation and then arrow back down to after *DONE* to continue more problems.

a. In your own words, write the conditions necessary for being able to multiply two matrices.

Answer: If the dimensions of the two matrices are $R_1 \times C_1$ and $R_2 \times C_2$, then they can only be multiplied if $C_1 = R_2$. In other words, the inner dimensions must be the same. The number of columns in the first matrix must match the number of rows in the second matrix.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

b. If matrix [*A*] has dimensions 3×2 and matrix [*B*] has dimensions 2×4, is it possible to multiply [*A*] times [*B*]? Explain your answer.

Answer: [*A*] can be multiplied by [*B*] since there are two columns in [*A*] and two rows in [*B*].

c. Using the matrices in part 1b, is it possible to multiply [*B*] times [*A*]? Explain your answer.

Answer: [*B*] cannot be multiplied by [*A*] since there are four columns in [*B*] and three rows in [*A*].

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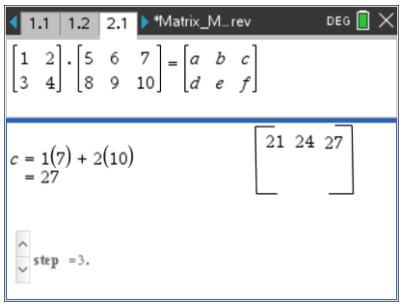

2. Click the ▲ arrow on the step slider to fill in the first few values in the answer matrix. Notice where the numbers are coming from in the two matrices being multiplied.

   a. Before going to step 3, predict the value of *c*. **27** Confirm your prediction.

      Answer: Step 3 gives 1(7) + 2(10) = 27.

   b. Repeat by predicting the values of *d*, *e*, and *f* before using the step slider to see the answers.

      Answer: *d* = 47, *e* = 54, *f* = 61

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

   c. If a 2 × 3 matrix is multiplied by a 3 × 5 matrix, what are the dimensions of the answer matrix?

      Answer: The answer matrix will have dimensions 2 × 5.

   d. Give an example of the dimensions of two matrices that, when multiplied, will result in an answer matrix with dimensions 3 × 4. Is your answer the only possible one?

      Answer: There are an infinite number of answers. The important conditions are that the outside dimensions are 3 and 4 and the inside dimensions are equal. For example, a matrix with dimensions 3 × 6 multiplied by a matrix with dimensions 6 × 4 will result in an answer matrix with dimensions 3 × 4.

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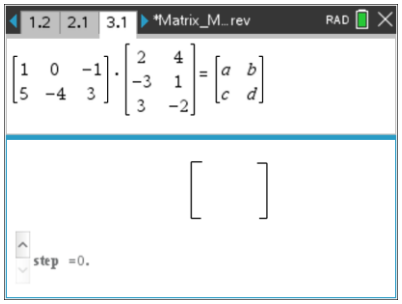


3. Click the ▲ arrow on the step slider to fill in the first few values in the answer matrix.
   a. Explain why the solution matrix is a 2 × 2 matrix.

      Answer: The first matrix is a 2 × 3, and the second is a 3 × 2. The outside dimensions are 2 × 2.

b. For *a*, *b*, *c*, and *d* in the solution matrix, show the work that results in these answers.

Answer:

$$\begin{bmatrix} a = 1\cdot 2 + 0\cdot(-3) - 1\cdot 3 & b = 1\cdot 4 + 0\cdot 1 - 1\cdot(-2) \\ c = 5\cdot 2 - 4\cdot(-3) + 3\cdot 3 & d = 5\cdot 4 - 4\cdot 1 + 3\cdot(-2) \end{bmatrix}$$

4. Josh, Jackie, and Jamilla have placed an order for lunch at a local fast-food restaurant. Josh ordered two cheeseburgers, two orders of fries, and a chocolate shake. Jackie ordered one cheeseburger and two chocolate shakes. Jamilla ordered one cheeseburger and one order of fries.

The information for each item follows:

Item	Cost	Calories	Fat
Cheeseburger	\$1	300	12
Fries	\$1	380	19
Shake	\$2	580	21

a. Set up a 3×3 matrix that represents each person's order $([A])$.

Answer:

$$[A] = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

b. Set up a 3×3 matrix that represents the cost, calories, and fat of the items $([B])$.

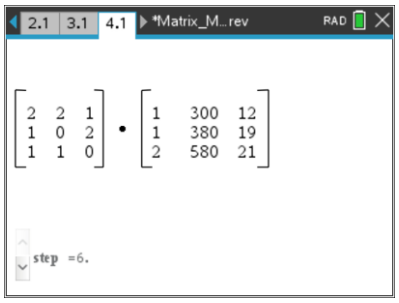
Answer:

$$[B] = \begin{bmatrix} 1 & 300 & 12 \\ 1 & 380 & 19 \\ 2 & 580 & 21 \end{bmatrix}$$

Move to page 4.1.

Click the ▲ arrow on the step slider to check your answers to parts 4a and 4b.

c. Multiply the two matrices together $([A]\cdot[B])$. Show your work.



Answer:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 300 & 12 \\ 1 & 380 & 19 \\ 2 & 580 & 21 \end{bmatrix} = \begin{bmatrix} 2\cdot1+2\cdot1+1\cdot2 & 2\cdot300+2\cdot380+1\cdot580 & 2\cdot12+2\cdot19+1\cdot21 \\ 1\cdot1+0\cdot1+2\cdot2 & 1\cdot300+0\cdot380+2\cdot580 & 1\cdot12+0\cdot19+2\cdot21 \\ 1\cdot1+1\cdot1+0\cdot2 & 1\cdot300+1\cdot380+0\cdot580 & 1\cdot12+1\cdot19+0\cdot21 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1940 & 83 \\ 5 & 1460 & 54 \\ 2 & 680 & 31 \end{bmatrix}$$

d. What do the numbers in the first column of the solution matrix represent?

Answer: The first column is the price for each person's meal.
Josh: \$6, Jackie: \$5, Jamilla: \$2

e. Who had the meal with the lowest number of calories?

Answer: Jamilla's meal had the lowest number of calories (680).

f. Josh's meal represents what percentage of the total fat of the group's meals?

Answer: Josh's meal represents $\frac{83}{83+54+31} = 0.494$ or 49.4% of the total fat of the group's meals.

5. In your own words, explain how to multiply two matrices.

Sample Answer: Each element in the answer matrix is found by summing the results of multiplying the elements in a row by the elements in a column. For example, if $[A]\cdot[B]=[C]$,

$c_{1,1} = a_{1,1}(b_{1,1}) + a_{1,2}(b_{2,1}) + a_{1,3}(b_{3,1}) + \ldots$
$c_{1,2} = a_{1,1}(b_{1,2}) + a_{1,2}(b_{2,2}) + a_{1,3}(b_{3,2}) + \ldots$
etc.
$c_{2,1} = a_{2,1}(b_{1,1}) + a_{2,2}(b_{2,1}) + a_{2,3}(b_{3,1}) + \ldots$
etc.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The conditions that must be met to allow two matrices to be multiplied.
- The process involved when multiplying two matrices.
- How to find the dimensions of an answer matrix.

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Note 1

Entire Document, *Screen Capture* or *Live Presenter*: If students experience difficulty with the syntax of any question, use the computer software or Live Presenter with TI-Nspire Navigator to demonstrate the correct syntax for the class.

Note 2

Question 1a, *Quick Poll*: Since this is an important concept in matrix multiplication, you could do a Quick Poll to ensure that students understand the concept before going on to do the actual multiplication. You might ask questions like 1b and 1c. The following are sample questions you can use:

1. Matrix $[A]$ is a 5×3 matrix, and Matrix $[B]$ is a 5×5 matrix. Is it possible to multiply $[A] \cdot [B]$? **(No)**

2. Is it possible to multiply $[B] \cdot [A]$? **(Yes)**

3. Is it possible to multiply $[B] \cdot [B]$? **(Yes)**

If students have difficulty with these questions, have them return to page 1.2 for more practice.

Note 3:

Question 2, *Quick Poll*: You could do a Quick Poll to ensure that students understand the concept of matrix multiplication. You might ask questions like 2c and 2d. The following are some sample questions you can use:

1. Matrix $[A]$ is a 5×3 matrix, and Matrix $[B]$ is a 3×8 matrix. What are the dimensions of the solution matrix to $[A] \cdot [B]$?
   a. 3×3
   b. 5×8
   c. 8×5
   d. not possible

2. Matrix $[A]$ is an $m \times n$ matrix, and Matrix $[B]$ is a $p \times m$ matrix. What are the dimensions of the solution matrix to $[B] \cdot [A]$?
   a. $n \times p$
   b. $m \times m$
   c. $p \times n$
   d. not possible