Looking Through the Lens – ID: 12379
By Irina Lyublinskaya

Topic: Light and Sound
- *Determine the positions and magnifications of images produced by converging and diverging lenses using ray diagrams.*
- *Determine the positions and magnifications of images produced by converging and diverging lenses using the thin lens equation and the magnification equation.*

Activity Overview
In this activity, students use simulated experiments to explore ray diagrams for converging and diverging thin lenses. Based on observations and data, students derive the lens equation and the magnification equation. They apply their understanding of how lenses work to real-world situations and problem solving.

Materials
To complete this activity, each student or student group will require the following:
- *TI-Nspire™ technology*
- *blank sheet of paper*
- *pen or pencil*

TI-Nspire Applications
Graphs & Geometry, Lists & Spreadsheet, Data & Statistics, Notes

Teacher Preparation
Before carrying out this activity, you should review with students the concepts of light refraction through curved surfaces, convex and concave lenses, the focal distance of a lens, and real and virtual images. Students should also have an understanding of the ideal model of a thin lens (i.e., when the thickness of a lens can be neglected because the light beam enters and leaves the lens at the same height).

- *The screenshots on pages 2–11 demonstrate expected student results. Refer to the screenshots on pages 12–14 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "12379" in the search box.***

Classroom Management
- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration during this activity:
- What is the relationship between power, curvature, and focal distance of a lens?
- How does the position of an object relative to the focal point affect the position and properties of an image for converging and diverging lenses?
- What is the lens equation?

The purpose of this activity is to allow students to explore ray diagrams for converging and diverging lenses. Based on these observations, students will derive the lens equation and the formula for lens magnification.

This activity consists of five problems. In the first problem, students explore the relationship between power, curvature, and focal distance for converging and diverging lenses. In the next two problems, students explore image formation for various positions of an object for both converging and diverging lenses. They use these observations to derive the lens equation. In the last two problems, students explore the relative sizes of an object and its image and derive the formula for the magnification of a lens. In each part of the activity, students apply their findings to problem solving.

Problem 1 – Power of a lens

Step 1: Students should open the file **PhyAct_12379_Lenses.tns**, read the first two pages, and then move to page 1.3, which shows simulations of a converging lens (top) and a diverging lens (bottom). Students should drag point *C* in each simulation to change the curvature of each lens and observe what happens to the focal distance and to the degree of bending of the light beams that pass through the lens. Students should press (ctrl) (tab) to move between lens simulations. Then, students should answer questions 1 and 2.

Q1. How does focal distance depend on the curvature of the lens?

A. *The larger the curvature of the lens is, the smaller the focal distance is.*

Q2. How does the shape of the lens affect the bending of the light beam? Which lens is converging and which one is diverging? How do you know?

A. *When the lens is biconvex (curves out on both sides), the light bends toward the optical axis, so the lens is converging. When the lens is biconcave (curves in on both sides), the light bends away from the optical axis, so the lens is diverging.*
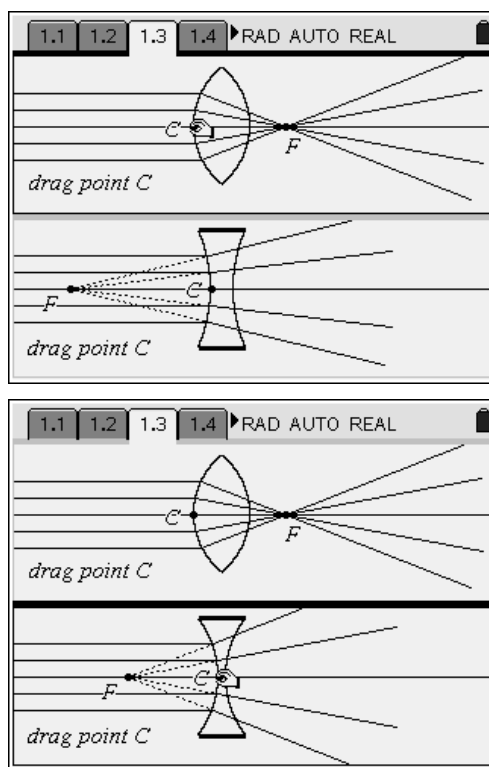
Step 2: Next, students should answer multiple-choice questions 3 and 4 on pages 1.5 and 1.6, and conceptual question 5 on page 1.7.

Q3. A lens has a focal distance of 10 cm. What is its power, in diopters?

A. *C; for a lens,* $P = \dfrac{1}{f}$, *where* P *is power in diopters and* f *is focal length in meters. Therefore,* $P = \dfrac{1}{f} = \dfrac{1}{0.1\,m} = 10\,D$. *Students can check their answers to multiple-choice questions (**Menu > Check Answer**) after they have answered them.*

Q4. What is the focal length in centimeters of a 100 D lens?

A. *B;* $f = \dfrac{1}{P} = \dfrac{1}{100\,D} = 0.01\,m = 1\,cm$

**Q5.** Explain why focusing on a distant object requires less curvature than focusing on a nearby object.

A. *The closer an object is to the lens, the more the rays reaching the lens diverge from one another. Bending these diverging rays together therefore requires a more curved lens.*
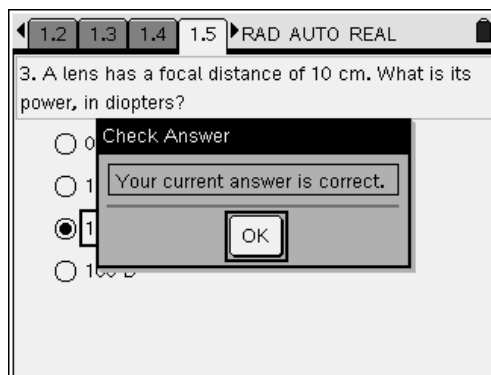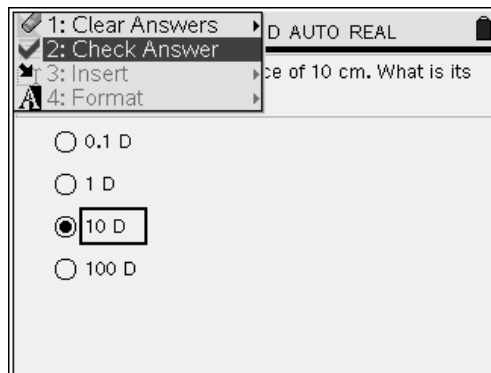
Problem 2 – Exploring ray diagrams for a converging lens

Step 1: Students should read page 2.1 and then move to page 2.2, where they can explore a ray diagram for a converging lens. (To maintain the accuracy of the simulation, the object should remain to the left of the focal point at all times.) The focus of the lens is kept constant.

Step 2: Students should drag point *O* to change the position of the object. They should study the position of the image, *I*, and the distance from the lens to the object, **dobj**, and to the image, **dimg**. Note: The positive direction for the *object* is to the left of the lens, and the positive direction for the *image* is to the right of the lens. Thus, when image is located on the same side as the object, the image is virtual, and **dimg** < 0.
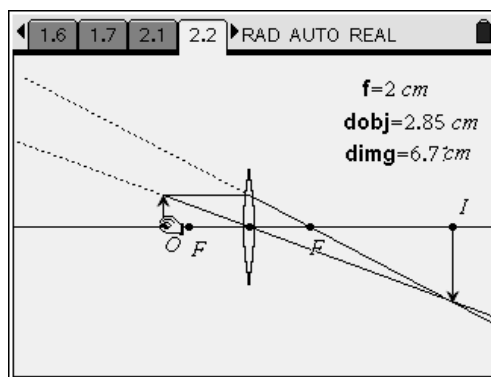
Step 3: Next, students should move to page 2.3. The distances (**dobj** and **dimg**) from page 2.2 are automatically captured in columns A and B in the *Lists & Spreadsheet* application on this page. Columns C and D contain the reciprocals of those distances (*obj_r* and *img_r*, respectively). Students should study the collected data and values of $\dfrac{1}{\textbf{dobj}}$

and $\dfrac{1}{\textbf{dimg}}$ and begin to infer characteristics of their relationship (for example, they may observe that the sum of these values stays constant).

|	A object	B image	C obj_r	D img_r
◆	=capture('	=capture('d	=1/object	=1/image
1	5.95	3.01266	0.168067	0.33193
2	5.9	3.02564	0.169492	0.33050
3	5.85	3.03896	0.17094	0.3290
4	5.75	3.06667	0.173913	0.32608
5	5.55	3.12676	0.18018	0.3198

C1 =0.16806722689076

Step 4: Next, students should move to page 2.4, which contains a *Data & Statistics* application. Students should use the application to make a plot of *img_r* vs. *obj_r*.
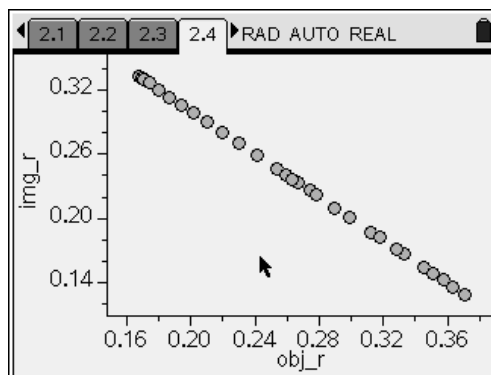
Step 5: The scatter plot should show a linear relationship. Students should use the **Linear Regression** tool (**Menu > Analyze > Regression > Show Linear (mx + b)**) to identify the best-fit equation for the data. (The best-fit equation should be $y = 0.5 - x$.) Next, students should answer questions 6 and 7.
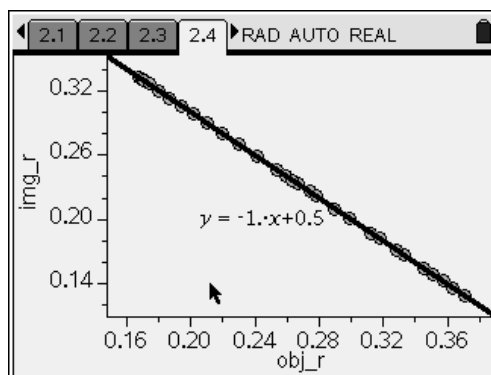
Q6. Describe all situations in which the image is real. Derive an equation describing the relationship between **f**, **dobj**, and **dimg** in these situations.

A. *The image is real when **dimg** > 0—that is, when the image is to the right of the lens. This happens when the object is at a distance larger than the focal distance from the lens. From the*

regression equation, $\dfrac{1}{dimg} = \dfrac{1}{2} - \dfrac{1}{dobj}$.

*Because **f** = 2, the equation relating* f*, **dobj**, and **dimg** is* $\dfrac{1}{dobj} + \dfrac{1}{dimg} = \dfrac{1}{f}$.

Q7. Describe all situations in which the image is virtual. Derive an equation describing the relationship between **f**, **dobj**, and **dimg** in these situations.

A. *The image is virtual when **dimg** < 0—that is, when image is to the left of the lens. This happens when the object is at a distance smaller than the focal distance from the lens.*

The equation $\dfrac{1}{dobj} + \dfrac{1}{dimg} = \dfrac{1}{f}$ *holds true for*

this situation as well. Note: If you wish, you may have students verify the lens equation for different focal distances. To do that, they should unlock the characteristics of point F *(using the **Attributes** menu), drag the point to vary the focal distance, and verify the lens equation for a different value of **f**. Before performing this test, students should clear the captured data on page 2.3.*
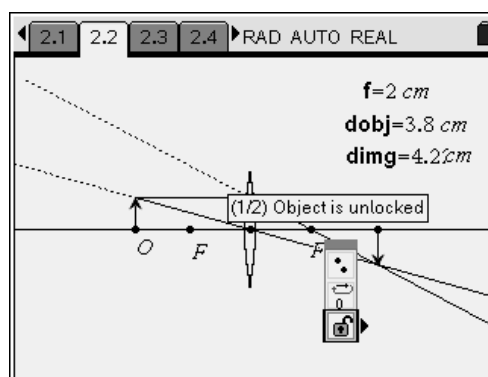
Step 6: Next, students should move to page 2.6 and solve question 8 using the lens equation they derived.

Q8. A lens has a focal length of 20 cm. It is positioned 15 cm from an object. How far from the lens is the image formed? Describe the image (real or virtual, enlarged or reduced, upright or inverted).

A. *Students should use the* Calculator *application on the bottom of the page to calculate the distance:* $\frac{1}{15} + \frac{1}{d_{img}} = \frac{1}{20}$, *so* $d_{img} = -60$ *cm.*

Since distance is negative, the image is virtual. It is farther away from the lens than the object is, so it is enlarged and upright.

| 2.3 | 2.4 | 2.5 | 2.6 | RAD AUTO REAL |

8. A lens has a focal length of 20 cm. It is positioned 15 cm from an object. How far from the lens is the image formed? Describe the image (real or virtual, enlarged or reduced, upright or inverted).

$$\frac{1}{20} - \frac{1}{15} \qquad \frac{-1}{60}$$

1/1

Problem 3 – Exploring ray diagrams for a diverging lens

Step 1: Students should read page 3.1 and move to page 3.2, where they can explore a ray diagram for a diverging lens. (To maintain the accuracy of the simulation, the object should remain to the left of the lens at all times.) The focus of the lens is kept constant.

Step 2: Students should drag point *O* to change the position of the object. They should observe the position of the image, *I*, and distances from the lens to the object, **dobj**, and to the image, **dimg**. Note: For the diverging lens the image is always virtual, so **dimg** < 0.

| 2.5 | 2.6 | 3.1 | 3.2 | RAD AUTO REAL |

f=-2 *cm*
dobj=5.7 *cm*
dimg=-1.48 *cm*

O   F I   F

Step 3: Next, students should move to page 3.3. The distances (**dobj** and **dimg**) from page 3.2 are automatically captured in columns A and B in the *Lists & Spreadsheet* application on this page. Columns C and D contain the reciprocals of those distances (*obj_r* and *img_r*, respectively). Students should study the collected data and values of $\frac{1}{dobj}$ and $\frac{1}{dimg}$ and begin to infer characteristics of their relationship.

| 2.6 | 3.1 | 3.2 | 3.3 | RAD AUTO REAL |

	A object	B image	C obj_r	D img_r
◆	=capture('d	=capture('d	=1/object	=1/image
1	4	-1.33333	1/4	-0.7
2	4.05	-1.33884	0.246914	-0.7469
3	4.1	-1.34426	0.243902	-0.74390
4	4.3	-1.36508	0.232558	-0.73255
5	4.5	-1.38462	0.222222	-0.72222

| A1 | =4 |

Step 4: Next, students should move to page 3.4, which contains a *Data & Statistics* application. Students should use the application to make a plot of *img_r* vs. *obj_r*, as they did in step 4 of problem 2.
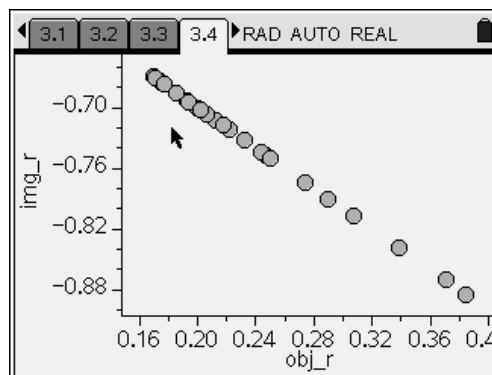
Step 5: The scatter plot should show a linear relationship. Students should use the **Linear Regression** tool to identify the best-fit equation for the data. (The best-fit equation should be $y = -0.5 - x$.) Next, students should answer question 9.
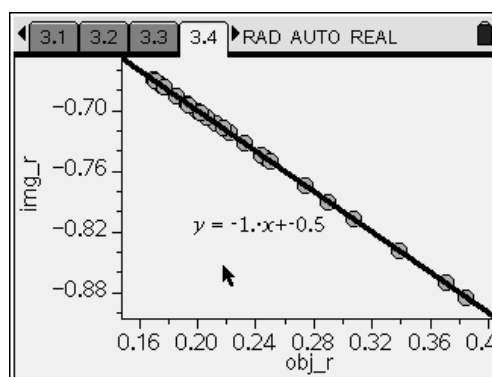
Q9. Does the relationship between **f**, **dobj**, and **dimg** hold true for the diverging lens?

A. *From the regression equation,*

$$\frac{1}{dimg} = -\frac{1}{2} - \frac{1}{dobj} \text{ . Because the lens is}$$

diverging, f = –2, so the final equation is

$$\frac{1}{dobj} + \frac{1}{dimg} = \frac{1}{f} \text{ , which is the same as in the}$$

case of the converging lens. Note: Students can verify the lens equation for different focal distances the same way that they did in question 7.

Step 6: Next, students should move to page 3.6 and solve the problem using the derived equation.

Q10. The object is now placed 20 cm in front of a diverging lens, which has a focal length of 5 cm. How far from the lens is the image? Describe the image (real or virtual, enlarged or reduced, upright or inverted).

 **A.** *Students should keep in mind that the focal distance for the diverging lens should be negative in the equation.* $\dfrac{1}{20} + \dfrac{1}{d_{img}} = -\dfrac{1}{5}$, *so*

    $d_{img} = -4$ *cm. Since distance is negative, the image is virtual. It is closer to the lens than the object, so it is reduced and upright.*
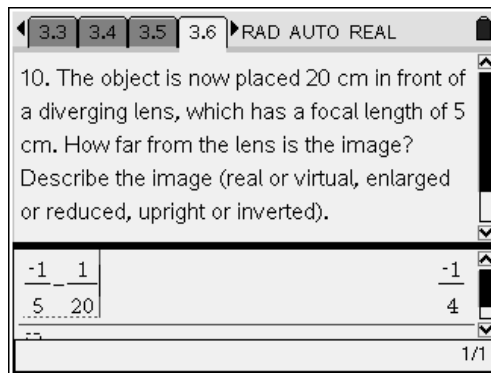
Problem 4 – Exploring the magnification of a converging lens

Step 1: Students should read the text on page 4.1 and then move to page 4.2, where they can explore a ray diagram for a converging lens. (To maintain the accuracy of the simulation, the object should remain to the left of the lens at all times.) The focus of the lens is kept constant.
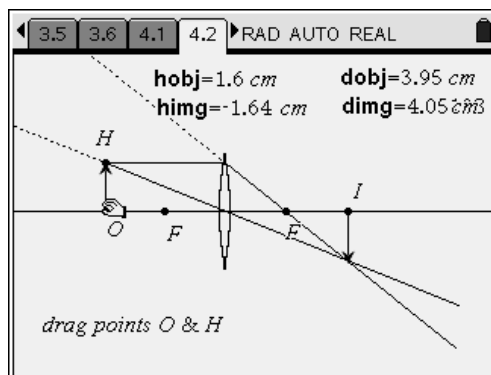
Step 2: Students should calculate magnification using the **Calculate** tool. To do this, they should first add a text box to page 4.2 (**Menu > Actions > Text**). In the text box, they should enter the expression a/b. They should then select the **Calculate** tool (**Menu > Actions > Calculate**). Once the **Calculate** tool is selected, students should click once on the text box they just created. When they are prompted to select *a*, they should click on the value of **himg**; when they are prompted to select *b*, they should click on the value of **hobj**. The value of the expression should appear on the screen. Students can click anywhere on the screen to place the calculated value at that location.
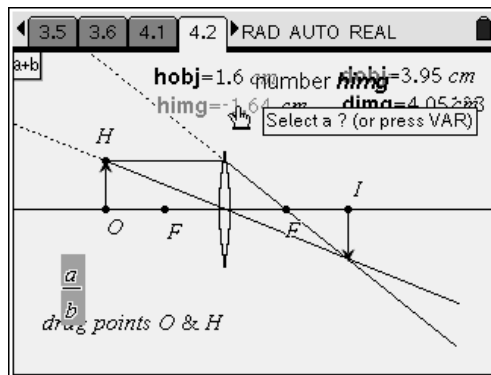
Step 3: Next, students should store the value they just calculated in the variable **M**. To do this, they should click once on the value, press ⟨store var⟩, select **Store Var**, and type *M.* They can press ⟨enter⟩ to complete the operation.
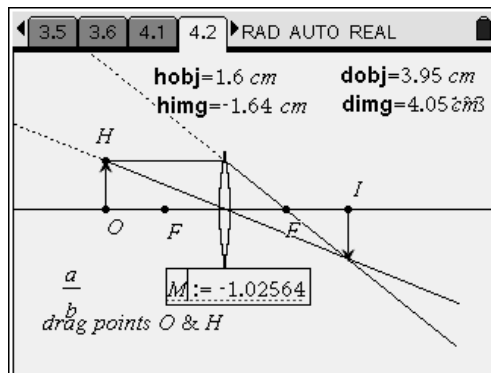
Step 4: Students should drag point *O* to change the position of the object and/or drag point *H* to change the height of the object. They should observe the values of magnification, **M**, in relation to the distances from the lens to the object, **dobj**, and to the image, **dimg**. They should record their observations for at least five different positions/heights of the object. A sample data set is shown in the table below.

dobj	dimg	M
1	–2	2
3	6	–2
4	4	–1
5	3.33	–0.67
6	3	–0.5
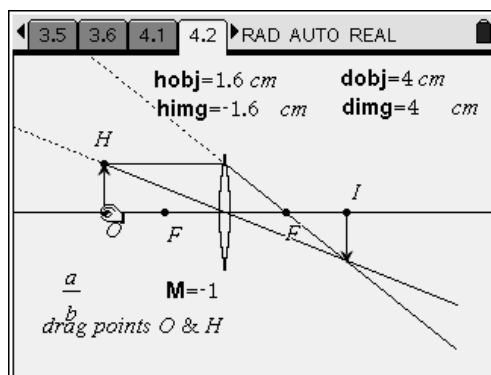
Step 5: Students should answer questions 11 and 12.

Q11. What is the mathematical relationship between **M**, **dobj**, **dimg**, **hobj**, and **himg**?

A. *Based on the data,* $M = -\dfrac{dimg}{dobj}$. *Thus,*

$\dfrac{dimg}{dobj} = -\dfrac{himg}{hobj}$. *This relationship also makes sense geometrically (similar triangles).*

Q12. A 5.0 cm tall object is positioned 15 cm in front of a lens, forming a real image 10 cm behind the lens. How tall is the image? What is the focal length of the lens?
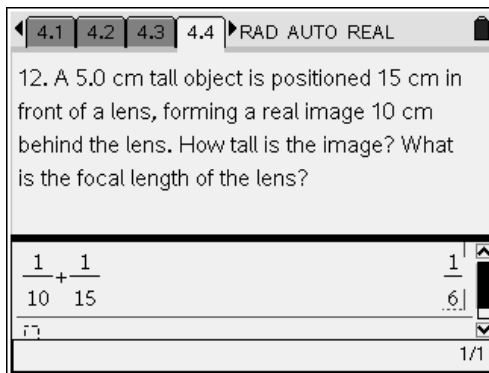
A. $M = -\dfrac{d_{img}}{d_{obj}} = -\dfrac{10}{15} = -\dfrac{2}{3}$. *Therefore,*

$h_2 = Mh_1 = -\dfrac{2}{3} \times 5 = -3.33 \text{ cm}$, *and the image is inverted. To find the focal length, use the lens formula:* $\dfrac{1}{f} = \dfrac{1}{d_{img}} + \dfrac{1}{d_{obj}} = \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{1}{6}$, *so*

$f = 6 \text{ cm.}$

Problem 5 – Exploring the magnification of a diverging lens

Step 1: Students should read the text on page 5.1 and then move to page 5.2, where they can explore a ray diagram for a diverging lens. (To maintain the accuracy of the simulation, the object should remain to the left of the lens at all times.) The focus of the lens is kept constant.

Step 2: Students should calculate the magnification of the lens using the **Calculate** tool, as they did in step 2 of problem 4. They should again store the magnification in variable **M**, as they did in step 3 of problem 4.
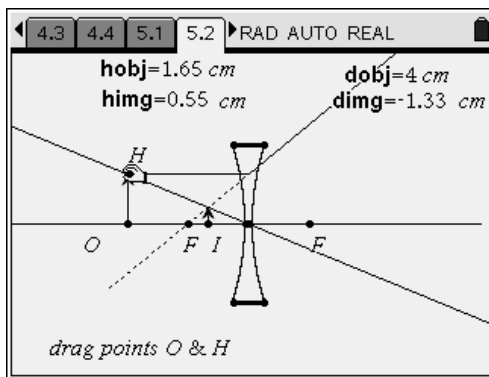
Step 3: Students should drag point *O* to change the position of the object and/or drag point *H* to change the height of the object. They should observe the values of magnification, **M**, in relation to the distances from the lens to the object, **dobj**, and to the image, **dimg**. They should record their observations for at least five different positions/heights of the object. A sample data set is shown in the table below.
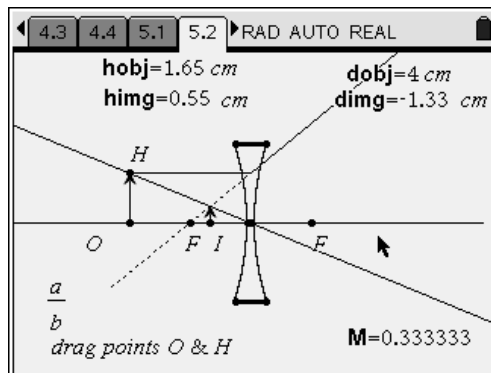
dobj	dimg	M
1	−0.67	0.67
2	−1	0.5
3	−1.2	0.4
4	−1.33	0.33
6	−1.5	0.25

Step 4: Students should answer questions 13 and 14.

Q13. Does the relationship between magnification and distance that you derived in question 11 hold true for diverging lenses?

  A. *Yes, it does. The triangles are similar in both cases.*

Q14. A 10.0 cm tall object is positioned 15 cm in front a lens, forming a virtual image 5 cm away from the lens. How tall is the image? What is the focal length of the lens?

  A. $M = -\dfrac{d_{obj}}{d_{img}} = -\dfrac{-5}{15} = \dfrac{1}{3}$, so $h_{img} = M h_{obj} = \dfrac{1}{3} \times 10 = 3.33 \ cm$. *The image is upright. To find*

  focal distance, use the lens formula: $\dfrac{1}{f} = \dfrac{1}{d_{img}} + \dfrac{1}{d_{obj}} = \dfrac{1}{15} + \dfrac{1}{-5} = \dfrac{-2}{15}$, *so*

  $f = -7.5 \ cm$.

Looking Through the Lens – ID: 12379

(Student)TI-Nspire File: *PhyAct_12379_Lenses.tns*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL	1.1 1.2 1.3 1.4 ▶RAD AUTO REAL	1.1 1.2 1.3 1.4 ▶RAD AUTO REAL
LOOKING THROUGH THE LENS **Physics** Optics	Lenses bend light rays such that they diverge or converge. Lenses have a "power." Power is measured in diopters and is given by the formula $P=\dfrac{1}{f}$, where *f* is the focal length of the lens. In this part of the activity, you will explore how the bending of a light beam depends on the focal length of a lens.	*drag point C* *drag point C*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL	1.2 1.3 1.4 1.5 ▶RAD AUTO REAL	1.3 1.4 1.5 1.6 ▶RAD AUTO REAL
1. How does focal distance depend on the curvature of the lens? 2. How does the shape of the lens affect the bending of the light beam? Which lens is converging and which one is diverging? How do you know?	3. A lens has a focal distance of 10 cm. What is its power, in diopters? ○ 0.1 D ○ 1 D ○ 10 D ○ 100 D	4. What is the focal length in centimeters of a 100 D lens? ○ 0.1 cm ○ 1 cm ○ 10 cm ○ 100 cm

1.4 1.5 1.6 1.7 ▶RAD AUTO REAL	1.5 1.6 1.7 2.1 ▶RAD AUTO REAL	1.6 1.7 2.1 2.2 ▶RAD AUTO REAL
5. Explain why focusing on a distant object requires less curvature than focusing on a nearby object.	Next, explore the properties of an image formed by a thin converging lens. The focal length of the lens on the next page is fixed. As you vary the distance to the object, **dobj**, observe changes in the distance from the lens to the image, **dimg**, and in the relative position of the image.	$f = 2\ cm$ $dobj = 4\ cm$ $dimg = 4\ cm$

1.7 2.1 2.2 2.3 ▶RAD AUTO REAL	2.1 2.2 2.3 2.4 ▶RAD AUTO REAL	2.2 2.3 2.4 2.5 ▶RAD AUTO REAL
A object / B image / C obj_r / D img_r ◆ =capture('(=capture('d =1/object =1/image 1 4 4. 1/4 0.2 2 3 4 5 A1 =4	Caption: <none> Click to add variable Click to add variable	6. Describe all situations in which the image is real. Derive an equation describing the relationship between **f**, **dobj**, and **dimg** in these situations. 7. Describe all situations in which the image is virtual. Derive an equation describing the relationship between **f**, **dobj**, and **dimg** in these situations.

◀ 2.3 | 2.4 | 2.5 | 2.6 ▶RAD AUTO REAL

8. A lens has a focal length of 20 cm. It is positioned 15 cm from an object. How far from the lens is the image formed? Describe the image (real or virtual, enlarged or reduced, upright or inverted).

0/99

◀ 2.4 | 2.5 | 2.6 | 3.1 ▶RAD AUTO REAL

Next, explore the properties of images formed by thin diverging lenses. The focal length of the lens on the next page is fixed. As you vary the distance to the object, **dobj**, observe changes in the distance from the lens to the image, **dimg**, and in the relative position of the image.

◀ 2.5 | 2.6 | 3.1 | 3.2 ▶RAD AUTO REAL

$f=-2$ cm
$dobj=4$ cm
$dimg=-1.33$ cm

O   F I   F

◀ 2.6 | 3.1 | 3.2 | 3.3 ▶RAD AUTO REAL

object	image	obj_r	img_r
=capture('d	=capture('d	=1/object	=1/image
4	-1.33333	1/4	-0.7

A1 =4

◀ 3.1 | 3.2 | 3.3 | 3.4 ▶RAD AUTO REAL

Caption: <none>

Click to add variable

◀ 3.2 | 3.3 | 3.4 | 3.5 ▶RAD AUTO REAL

9. Does the relationship between **f, dobj**, and **dimg** hold true for the diverging lens?

◀ 3.3 | 3.4 | 3.5 | 3.6 ▶RAD AUTO REAL

10. The object is now placed 20 cm in front of a diverging lens, which has a focal length of 5 cm. How far from the lens is the image? Describe the image (real or virtual, enlarged or reduced, upright or inverted).

0/99

◀ 3.4 | 3.5 | 3.6 | 4.1 ▶RAD AUTO REAL

Next, you will explore the magnification of lenses. The magnification, **M**, is given by $M=\dfrac{himg}{hobj}$, where **himg** and **hobj** are the heights of the image and the object, respectively. Explore the simulation for the converging lens on the next page.

◀ 3.5 | 3.6 | 4.1 | 4.2 ▶RAD AUTO REAL

$hobj=1.6$ cm $dobj=6$ cm
$himg=-0.8$ cm $dimg=3$ cm

H
O   F   F   I

drag points O & H

◀ 3.6 | 4.1 | 4.2 | 4.3 ▶RAD AUTO REAL

11. What is the mathematical relationship between **M, dobj, dimg, hobj**, and **himg**?

◀ 4.1 | 4.2 | 4.3 | 4.4 ▶RAD AUTO REAL

12. A 5.0 cm tall object is positioned 15 cm in front of a lens, forming a real image 10 cm behind the lens. How tall is the image? What is the focal length of the lens?

0/99

◀ 4.2 | 4.3 | 4.4 | 5.1 ▶RAD AUTO REAL

Next, you will explore the magnification of diverging lenses. Explore the simulation on the next page.

| 4.3 | 4.4 | 5.1 | 5.2 ▶ RAD AUTO REAL |

hobj=2 *cm*   **dobj**=4 *cm*
himg=0.67 *cm*   **dimg**=-1.33 *cm*

H

O   *F   I   F*

drag points O & H

| 4.4 | 5.1 | 5.2 | 5.3 ▶ RAD AUTO REAL |

13. Does the relationship between magnification and distance that you derived in question 11 hold true for diverging lenses?

| 5.1 | 5.2 | 5.3 | 5.4 | RAD AUTO REAL |

14. A 10.0 cm tall object is positioned 15 cm in front a lens, forming a virtual image 5 cm away from the lens. How tall is the image? What is the focal length of the lens?

⬚

0/99