

Activity 8

Objectives

- Use the CellSheet™ App to graph vectors
- Find the sum of two vectors

Visualizing Vectors

Introduction

Vectors are two-dimensional and they have magnitude and direction. The magnitude may be force or some other measured quantity while the direction is an angle, θ .

A vector in standard position begins at the origin and extends in a direction, θ for r units (magnitude). When vectors are defined using r and θ , they are said to be in *polar* form. A point that is r units in a direction θ may also be defined by an ordered pair (x, y) . Vectors defined in terms of x and y are in *rectangular* form.

In this activity, you will use the CellSheet application to visualize vectors as well as find the sum of two vectors.

To convert a vector from polar to rectangular coordinates:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

To convert a vector from rectangular to polar coordinates:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right)$$

Problem

Vectors are often defined by an ordered pair (r, θ) in their polar form.

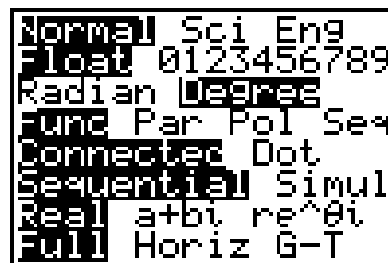
$V_1 = (5, 120^\circ)$ = Vector 1 has a magnitude of 5 and a direction of 120°

$V_2 = (3, 20^\circ)$ = Vector 2 has a magnitude of 3 and a direction 20°

Graph the vectors V_1 and V_2 and find and graph their sum, $V_1 + V_2$.

Exploration

1. Press **[MODE]** and select **Degree**. Press **[ENTER]**. Press **[2nd]** **[QUIT]** to exit **[MODE]** settings.
2. Create a new spreadsheet in the CellSheet™ App, and name it **VECTORS**.

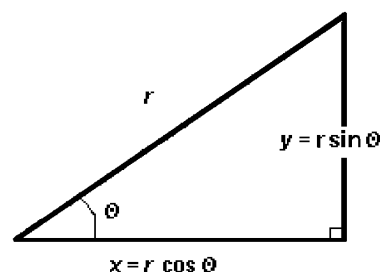


3. Enter the headings in cells A1, A2, A4, and A5 as shown.
4. The magnitude of V_1 is 5 and the direction is 120° , so enter 5 in cell B1 and 120 in cell B2.

| VECT | A | B | C |
|------|-------|-----|---|
| 1 | MAGN | 5 | |
| 2 | DIREC | 120 | |
| 3 | | | |
| 4 | MAGN | | |
| 5 | DIREC | | |
| 6 | | | |

B3: [Menu]

To graph the vector on a coordinate grid, you need to convert the polar coordinates to the rectangular coordinates. That is, you want to define the vector in terms of x and y instead of r and θ . From the chart in the introduction, you saw that you can do this using right triangle trigonometry.



5. a. Enter "X" in cell C1.
b. Enter "Y" in cell D1.
6. Enter 0 in cells C2 and D2 to define the end of the vector at the origin (0, 0).

| VECT | B | C | D |
|------|-----|---|---|
| 1 | 5X | Y | |
| 2 | 120 | 0 | 0 |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

D3: [Menu]

7. In cell C3, enter the formula **=B1*cos(B2)**. This will be the x -component of the vector.

What is the numerical value of the x -component? _____.

| VECT | B | C | D |
|------|-----|---|---|
| 1 | 5X | Y | |
| 2 | 120 | 0 | 0 |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |

C3: =B1*cos(B2)

8. In cell D3, enter the formula $=B1*\sin(B2)$. This will be the y -component of the vector.

What is the numerical value of the y -component? _____.

| VECT | E | C | D |
|-----------------|-----|------|---|
| 1 | 5% | | Y |
| 2 | 120 | 0 | 0 |
| 3 | | -2.5 | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| D3: =B1*sin(B2) | | | |

The coordinates of V_1 in standard position are $(0, 0)$ and $(-2.5, 4.3301)$. The graph of the vector uses the magnitude (length of the vector), and the direction (angle).

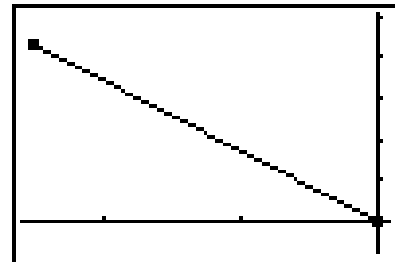
9. To graph the vector, select **Menu > Charts > Line**. Set the **XRange** to **C2:C3** and the **YRange** to **D2:D3**.

```

LINE CHART
XRange: C2:C3
YRange1: D2:D3
YRange2:
YRange3:
Title:
AxisOn AxesOff
DrawFit Draw

```

10. Press **ENTER** until the graph appears.



With a direction of 120° , the vector is in the second quadrant, about 60° above the horizontal axis or 30° to the left of the vertical axis. If the angle does not look like it is 60° above the horizontal, it is because the scales on the two axes are different.

The magnitude is relative to the grid, so 5 would appear reasonable.

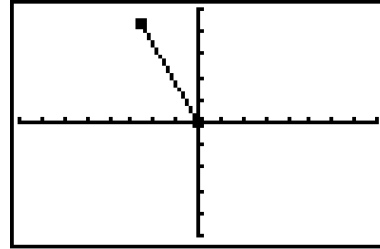
11. To make the figure more nearly approximate what would be manually drawn on rectangular coordinate grid paper, change the **LINE WINDOW** settings to match those shown. Highlight **Save** before pressing **ENTER** so that the settings will be retained by the handheld.

```

LINE WINDOW
Xmin=-7
Xmax=7
Xscl=1
Ymin=-5
Ymax=5
Yscl=1
Draw Save

```

These settings approximate the proportion of “squares” that would be visible in the area defined by the dimensions of the handheld’s screen.



12. Press **2nd** [QUIT] to return to the CellSheet™ App and enter the information shown for vector V_2 .

| VECT | A | B | C |
|------|-------|-----|------|
| 1 | MAGN | 5% | |
| 2 | DIREC | 120 | 0 |
| 3 | | | -2.5 |
| 4 | MAGN | 3 | |
| 5 | DIREC | 20 | |
| 6 | | | |

B6: [Menu]

13. a. Enter 0 in cell C4 and cell D4 to represent the points of origin (0, 0).
 b. Enter $=B4*\cos(B5)$ in cell C5 and $=B4*\sin(B5)$ in cell D5 to plot the x- and y- components of V_2 .

The x-component of V_2 is _____.

The y-component of V_2 is _____.

| VECT | B | C | D |
|------|-----|--------|--------|
| 1 | 5% | | |
| 2 | 120 | 0 | 0 |
| 3 | | -2.5 | 4.3301 |
| 4 | 3 | 0 | 0 |
| 5 | 20 | 2.8191 | |
| 6 | | | |

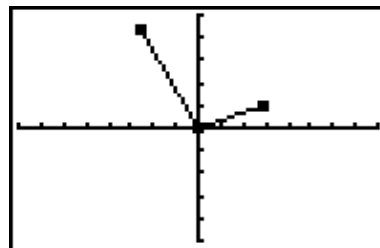
D5: =B4*sin(B5)

14. To graph the two vectors, select **Menu > Charts > Line**. Set the **XRange** to C2:C5 and the **YRange** to D2:D5. Be certain to highlight **Draw** rather than **DrawFit** so that the saved window settings will be used.

```

LINE CHART
XRange: C2:C5
YRange1: D2:D5
YRange2:
YRange3:
Title:
AxesOn  AxesOff
DrawFit  Draw
    
```

The two vectors are shown here.



15. The second vector is shorter than the first and is in the first quadrant. Look back at the polar coordinates of the two vectors.

V_1 has a magnitude of _____ and a direction of _____.

V_2 has a magnitude of _____ and a direction of _____.

Do the vectors presented seem to match these coordinates? Explain.

The sum of the two vectors is called the *resultant* of two vectors. Geometrically, the resultant is the diagonal of a parallelogram made by the two vectors.

To add two vectors (r_1, θ_1) and (r_2, θ_2) , write the two vectors in their rectangular form, in terms of x and y , (x_1, y_1) and (x_2, y_2) , and then add the two ordered pairs. The result is $(x_1 + x_2, y_1 + y_2)$. Notice that the sum is found algebraically using ordered pairs defined in terms of x and y , not r and θ .

16. a. Return to the CellSheet™ App by pressing [2nd] [QUIT]. The sum of the two vectors will be computed in cells C6, D6, C7, and D7.

b. Enter $=C2+C4$ in cell C6.

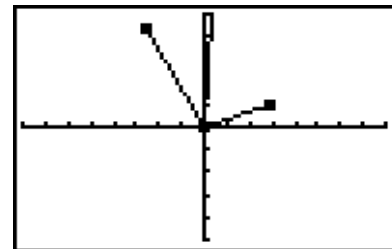
17. Copy that formula into cells D6, C7, and D7.

| VECT | B | C | D |
|-------|-----|--------------|--------|
| 2 | 120 | 0 | 0 |
| 3 | | -2.5 | 4.3301 |
| 4 | 3 | 0 | 0 |
| 5 | 20 | 2.8191 | 1.0261 |
| 6 | | 0 | |
| 7 | | | |
| D6:C7 | | [Faste] Menu | |

The two ordered pairs that define the resultant of the two vectors are in cells (C6, D6) and (C7, D7). The ordered pairs are (0, 0) and (0.31908, 5.3562).

18. To graph the two vectors and their resultant, select **Menu > Charts > Line**. Set the **XRange** to **C2:C7** and the **YRange** to **D2:D7**.

The graph of the two vectors and their sum (the resultant) is shown. However, the resultant is not easily visible with these window settings. You can again change the window settings to make the graph easier to interpret.



To optimize the benefits of identifying the new settings, these observations can inform the decision:

- a. the two vectors and the resultant all appear above the x -axis.
- b. the resultant is nearly superimposed on the y -axis; and
- c. the resultant appears to go beyond the top of the screen.

The settings shown can improve the readability of the display of the previous graph.

```

LINE WINDOW
Xmin=-8
Xmax=8
Xscl=1
Ymin=-.5
Ymax=7.5
Yscl=1
Draw Save

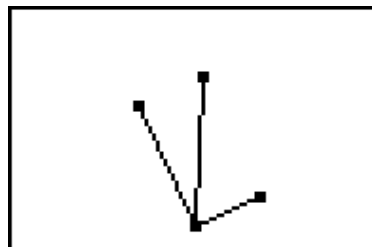
```

```

LINE CHART
XRange:C2:C7
YRange1:D2:D7
YRange2:
YRange3:
Title:
AxesOn AxesOff
DrawFit Draw

```

The graph now appears clearly without interference created by the axes.



19. You can now connect the endpoints of the two vectors and the resultant.

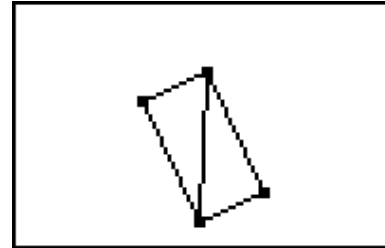
To connect the endpoints, you need to insert their x - and y -coordinates in the next available cells of the spreadsheet. The coordinates of the endpoints of V_1 , V_2 , and the resultant have been entered in columns C and D of rows 8, 9, and 10 respectively.

| VECT | E | C | D |
|------|---|--------|-------------|
| 8 | | -2.5 | 4.3301 |
| 9 | | .31908 | 5.3562 |
| 10 | | 2.8191 | 1.0261 |
| 11 | | | |
| 12 | | | |
| 13 | | | |
| D13: | | | Menu |

To display the complete figure, change the **XRange** and **YRange1** settings as shown.

```
LINE CHART
XRange:C2:C10
YRange1:D2:D10
YRange2:
YRange3:
Title:
AxesOn  AxesOff 
DrawFit  Draw 
```

The graph demonstrates that the resultant of V_1 and V_2 is the diagonal of a parallelogram having sides defined by the magnitude and direction of V_1 and V_2 .



Student Worksheet

Name _____

Date _____

Reviewing Concepts

1. A vector is a two-_____ quantity. It has _____ and _____.
A vector in standard position originates at the _____.
2. A vector defined in terms of r and θ can be _____ into components. The x -component is found using the formula _____. The y -component is found using the formula _____.
3. Vectors defined by r and θ are in _____ form. Vectors defined by x and y are in _____ form.
4. The graph of a vector in standard position is a line segment that begins at the _____ and extends _____ units in some direction _____.
5. In this activity, the vector V_1 has a magnitude of _____ and a direction of _____. The graph of V_1 is a line segment in the _____ quadrant of a coordinate grid. Vector V_2 has a magnitude of _____ and a direction of _____. The graph of V_2 is a line segment in the _____ quadrant.
6. The resultant of two vectors is the _____ of the two vectors. Graphically, the resultant $V_1 + V_2$, is a _____ of a parallelogram whose sides are V_1 and V_2 .

Solving the Problem

7. Complete the table.

| | r | θ | x | y |
|-----------------|-----|----------|-----|-----|
| Vector One | | | | |
| Vector Two | | | | |
| Sum (Resultant) | | | | |

Analyzing the Data

8. Explain why $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$
but $(r_1, \theta_1) + (r_2, \theta_2) \neq (r_1 + r_2, \theta_1 + \theta_2)$.

Extending the Activity

1. Graph V_3 , V_4 and $V_3 + V_4$.

$$V_3 = (8, 200^\circ)$$

$$V_4 = (5, 80^\circ)$$

2. Vectors are sometimes used to define a force. For example, let $F_1 = (8, 32^\circ)$ define a force of 8 pounds in a direction of 32° and let $F_2 = (6, 83^\circ)$ define a force of 6 pounds in a direction of 83° . Graph the two vectors and their sum. Explain the result in terms of the original vectors.
3. Explain why the resultant of these two vectors is larger than either one of the two but less than the sum of the two.

Teacher Notes



Activity 8

Visualizing Vectors

Preparation

Vectors are often felt, and not seen. You can feel the results of someone pushing you. You can feel the results of a crosswind as you drive over the top of a hill. This activity will help students visualize the two-dimensional aspect of a vector by allowing them to graph vectors and to find and graph the sum of two vectors. Students need to understand the concept of vector components before starting this activity. The formulas for resolving vectors and the formulas for converting between polar and rectangular forms are given, but it is best if the students have practiced with the formulas before doing the activity.

Elicit Questions

An open container of water is tied to two ropes, each held on the end by a strong man. On the count of three, the two men begin to pull on the ropes in opposite directions. What happens to the water and the container as the two men pull? Explain.

Management

Have students make a sketch of the graphs of each vector. This practice helps many students visualize the vectors better and to approximate angles and lengths.

Objectives

- Use the CellSheet™ App to graph vectors
- Find the sum of two vectors

Materials

- TI-84 Plus/TI-83 Plus

Time

- 60 minutes

Answers to Exploration Questions

7. -2.5
8. About 4.3301
13. b. x -component of $V_2 = 2.8191$; y -component of V_2 is 1.0261
15. 5, 120° ; 3, 20° . Answers may vary. Students should acknowledge that V_1 is appropriately longer than V_2 . In addition V_1 , with a direction of 120° should be in the second quadrant, while V_2 should be in the first quadrant with a direction of 20° .

Answers to the Student Worksheet

Reviewing Concepts

1. Dimensional; magnitude; direction; origin
2. Resolved; $x = r \cos \theta$; $y = r \sin \theta$
3. Polar; rectangular
4. Origin; r ; θ
5. 5; 120° ; second; 3; 20° ; first
6. Sum; diagonal

Solving the Problem

7.

| | r | θ | x | y |
|-----------------|-----|----------|---------|--------|
| Vector One | 5 | 120 | -2.5 | 4.3301 |
| Vector Two | 3 | 20 | 2.8191 | 1.0261 |
| Sum (Resultant) | | | 0.31908 | 5.3562 |

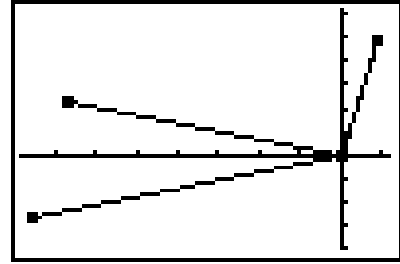
Analyzing the Data

8. Adding quantities in the same direction (either along the horizontal for x -values or the vertical for y -values) makes sense and is mathematically sound.

Adding r -values that are not in the same direction is not mathematically sound. The first sum is a property of the rectangular coordinate system. The second sum is not a property of polar coordinates. A counterexample proves that conjecture: $(1, 180^\circ) + (1, 0^\circ) \neq (2, 180^\circ)$.

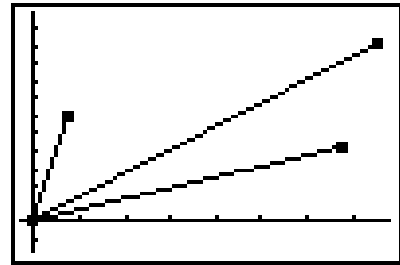
Extending the Activity

1. $V_3 = (8, 200^\circ)$
 $V_4 = (5, 80^\circ)$



2. The result of adding F_1 and F_2 is about $(7.52, 10.19)$.

This (x, y) ordered pair can be translated to (r, θ) using the two formulas $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. This means that the result of adding a force of 8 pounds in a direction of 32° to another force of 6 pounds in a direction of 83° is a force of 12.67 pounds in a direction of 53.6° .



3. If the two vectors were in the same directions, the resultant would be the sum of the two vectors. As the angle between the two vectors increases, the two vectors begin to take away from each other. That is, they pull against themselves to some degree. If the two vectors were in opposite directions, the resultant would be in the direction of the larger vector and the length of the resultant would be the difference between the two forces.