EXPLORATIONS

Activity 10

From a Distance... You Can See It!

Teacher Notes

Concepts

- Midpoint between two points
- Distance between two points
- Pythagorean Theorem

Calculator Skills

- Entering fractions: [A b/c]
- Setting decimal places: [2nd] [FIX]
- Using the [x^2] key

Materials

- TI-30X IIS
- Student Activity pages (p. 97-100)
- 3 x 5 index cards
- Coordinate grid paper (centimeter paper)

Objective

- In this activity, students will find distances between points using common fraction and decimal calculations with the concepts of midpoint and distance. They also will learn to solve problems using the Pythagorean Theorem and by finding the diagonal distance in a rectangular prism.

Topics Covered

- Translating between synthetic and coordinate representations
- Deducing properties of figures using coordinates
- Using coordinates in problem solving

Introduction

Construct a triangle on rectangular grid paper with vertices at F(0,0), G(12,0), and H(8,6). Estimate the location of the midpoint of side FH and call it point M. Estimate the location of the midpoint of side GH and call it point N. Connect MN with a line segment. What is the length of segment MN?

Investigation

1. Draw line segment $\overline{ST}$. Locate and label three midpoints as follows: Let R be the midpoint of $\overline{ST}$. Let Q be the midpoint of $\overline{RT}$ and P be the midpoint of $\overline{QT}$. The length of $\overline{ST}$ = 38 ½ centimeters.

2. Use the overhead calculator to find the length of $\overline{RT}$. Continue this procedure to find the lengths of $\overline{QT}$ *(9 5/8)* and $\overline{PT}$ *(4 13/16)*.

Press:	The calculator shows:
[CLEAR] 38 [A b/c] 1 [A b/c] 2 [ENTER]	38⌟1⌟2 38⌴1/2 DEG
[÷] 2 [ENTER]	Ans/2 19⌴1/4 DEG

3. On coordinate grid paper, locate and plot the points A(1,-2), B(1,7), C(5,7) and D(7,-2). Connect the points with line segments in order from A to B, B to C, C to D, and D to A. The geometric shape should be a trapezoid.

4. Find the midpoints for $\overline{AB}$, $\overline{BC}$, and $\overline{DA}$. Label these points M, E, and T respectively.

5. Finding the midpoint R of $\overline{CD}$ is more difficult. To find the midpoint for any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, use the midpoint formula:
$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

   Use the overhead calculator to find the midpoint R of segment $\overline{CD}$. Therefore, the coordinates of the midpoint R are (6, 2.5).

Press:	The calculator shows:
[CLEAR] [(] 5 [+] 7 [)] [÷] 2 [ENTER]	(5 + 7)/2 6 DEG
[(] 7 [+] [(-)] 2 [)] [÷] 2 [ENTER]	(7 + -2) 2.5 DEG

6. Find the length of $\overline{CD}$ above, using the distance formula:

   $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

   Use C(5, 7) and D(7, -2). Display the result with two decimal places.

Press:	The calculator shows:
[CLEAR] [2nd] [√‾] [(] 7 [−] 5 [)] [x^2] [+] [(] [(−)] 2 [−] 7 [)] [x^2] [)] [ENTER]	√ ((7−5)² + (-2→ 9.219544457 DEG
[2nd] [FIX] ▶ ▶ ▶	F 0 1 <u>2</u> 3 4 5 6 7 8 9 FIX DEG
[ENTER]	√ ((7−5)² + (-2→ 9.22 DEG

Wrap-Up

- Assign Student Activity Part 1 and Student Activity Part 2 as individual work and then have students discuss their answers in pairs.
- Use Student Activity Part 3 as a group activity to "discover" or reinforce the Pythagorean Theorem.
- Make sure students set the calculator to floating-decimal format at the end of this activity.

Extensions

The line segment $\overline{MN}$ in the Introduction is called the *median.* Ask students how they determined the estimates of the location of the midpoints. What is the length of the median? Why is this segment called the median? How many different trapezoids can you construct whose bases are odd numbers with height of 6 inches and whose area is 48 square inches?

Solutions Part 1

Draw line segment $\overline{LM}$ with midpoint T. Point X is the midpoint of segment $\overline{LT}$, and Y is the midpoint of segment $\overline{TM}$. Point Z is the midpoint of segment $\overline{LX}$. The distance from L to M is 42 1/4 centimeters. Express your answers as mixed fractions.

1. The distance TM is *21 1/8* centimeters.
2. The distance TX is *10 9/16* centimeters.
3. The distance YM is *10 9/16* centimeters.
4. The distance LZ is *5 9/32* centimeters.
5. The distance TZ is *15 27/32* centimeters.
6. The distance ZY is *26 13/32* centimeters.

Suppose that the distance from Z to Y is 18 3/4 centimeters. Express your answers as mixed fractions.

7. The distance ZT is *11 1/4* centimeters.
8. The distance LY is *22 1/2* centimeters.
9. The distance LM is *30* centimeters.
10. The distance YM is *7 1/2* centimeters.

Name all other segments that are the same length as $\overline{YM}$.

$\overline{TY}$, $\overline{XT}$, $\overline{LX}$

Solutions Part 2

Use the trapezoid from before with the points A(1,-2), B(1,7), C(5,7), and D(7,-2). Label the midpoints for $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ as points M, E, R, and T, respectively. Use the distance formula and your calculator to solve the problems below. Write your answers correct to three decimal places.

1. Find the distance AC. *9.849*
2. Find the distance BD. *10.817*
3. Find the distance MR. *5.000*
4. Find the distance ET. *9.055*
5. Find the distance MC. *6.021*
6. Find the distance BT. *9.487*
7. Find the perimeter of ABCD. *28.220* *AB = 9*

   BC = 4

   CD = 9.220

   DA = 6

8. Find the perimeter of MERT. *20.664* *ME = 4.924*
ER = 5.408
RT = 4.924
TM = 5.408

9. Find the ratio $\frac{\text{Perimeter of ABCD}}{\text{Perimeter of MERT}}$. *1.366*

10. How can you explain the answer for #9?

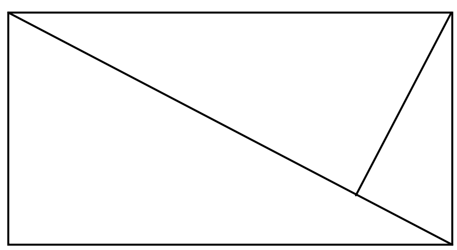
The perimeter of the trapezoid is 36.69% larger than the perimeter of the quadrilateral connecting the midpoints of the sides.

Solutions Part 3

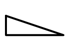
1. Cut a 3" x 5" card along one diagonal. How do the two right triangles compare?

   a. Draw an altitude from the right angle to the hypotenuse of one of the right triangles. Now cut along the altitude to create two triangles. Use the diagram below as a guide. You should now have three triangles. How do the three triangles compare?

   They are similar triangles



   b. Use a ruler to measure the lengths of the sides of the small triangle, the medium, triangle, and the large triangle. Record the measures in the table below, then use your calculator to find the squares of each of the measures. Record these calculated values in the table. (The answers will vary)

	Short Leg	Long Leg	Hypotenuse	(Short leg)^2	(Long Leg)^2	(Hypotenuse)^2
Small						
Medium						
Large						

   c. Describe the relationships that you observe between the legs and the hypotenuse in the table.

   The measures in the table should confirm the conclusion of the Pythagorean Theorem. That is, the sum of the squares of the short leg and the long leg should be equal to the square of the hypotenuse.

2. An antenna used to transmit television signals is 235 feet tall. Three support cables, each of which measure 205 feet, are attached to the antenna at a point that is 183 feet above the ground. How far away from the base of the antenna will the ends of the three support cables be when attached to the ground?

   The distance away from the antenna should be $\sqrt{205^2 - 183^2}$ *= 92.39 feet*.

3. The length *d* of the diagonal of any rectangular solid can be found by the formula $d = \sqrt{\text{length}^2 + \text{width}^2 + \text{height}^2}$. Why does this formula work?

   The formula works because it is an extension of the Pythagorean Theorem in three dimensions. (Answers may vary)

4. Jacqueline wishes to pack a 3 ½ - foot umbrella in a rectangular suitcase that measures 3 ft x 2 ft x ¾ ft. Is this possible? Why or why not?

   Yes! The maximum distance is the diagonal distance $\sqrt{2^2 + 3^2}$ *= 3.606 feet.*

Student Activity 10

Name ______________________________

Date ______________________________

Geometry and Measurement—From a Distance…You Can See It!

Objective: *In this activity, you will find the distance between points using common fraction and decimal calculations with the concepts of midpoint and distance. You also will solve problems using the Pythagorean Theorem and by finding the diagonal distance in a rectangular prism.*

Part 1: Finding Distances Between Points

Draw line segment $\overline{LM}$ with midpoint T. Point X is the midpoint of segment $\overline{LT}$, and Y is the midpoint of segment $\overline{TM}$. Point Z is the midpoint of segment $\overline{LX}$. The distance from L to M is 42 ¼ centimeters. Express your answers as mixed fractions.

1. The distance TM is ________ centimeters.
2. The distance TX is ________ centimeters.
3. The distance YM is ________ centimeters.
4. The distance LZ is ________ centimeters.
5. The distance TZ is ________ centimeters.
6. The distance ZY is ________ centimeters.

Suppose that the distance from Z to Y is 18 ¾ centimeters. Express your answers as mixed fractions.

7. The distance ZT is ________ centimeters.
8. The distance LY is ________ centimeters.
9. The distance LM is ________ centimeters.

10. The distance YM is ___________ centimeters.

11. Name all other segments that are the same length as $\overline{YM}$.

Part 2: Finding Measurements on a Trapezoid

Use the trapezoid from before with the points A(1,-2), B(1,7), C(5,7), and D(7,-2). Label the midpoints for $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ as points M, E, R, and T, respectively. Use the distance formula and your calculator to solve the problems below. Write your answers correct to three decimal places.

1. Find the distance AC.

2. Find the distance BD.

3. Find the distance MR.

4. Find the distance ET.

5. Find the distance MC.

6. Find the distance BT.

7. Find the perimeter of ABCD.

8. Find the perimeter of MERT.

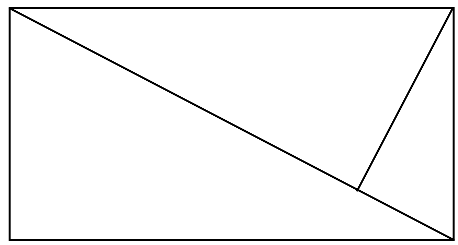
9. Find the ratio $\frac{\text{Perimeter of ABCD}}{\text{Perimeter of MERT}}$.

10. How can you explain the answer for #9?

Part 3: Working with Triangles

1. Cut a 3″ x 5″ card along one diagonal. How do the two right triangles compare?

   a. Draw an altitude from the right angle to the hypotenuse of one of the right triangles. Now cut along the altitude to create two triangles. Use the diagram below as a guide. You should now have three triangles. How do the three triangles compare?



   b. Use a ruler to measure the lengths of the sides of the small triangle, the medium triangle, and the large triangle. Record the measures in the table below, and then use your calculator to find the squares of each of the measures. Record these calculated values in the table.

	Short Leg	Long Leg	Hypotenuse	(Short leg)^2	(Long Leg)^2	(Hypotenuse)^2
Small						
Medium						
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   c. Describe the relationships that you observe between the legs and the hypotenuse in the table.

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3. The length d of the diagonal of any rectangular solid can be found by the formula $d = \sqrt{\text{length}^2 + \text{width}^2 + \text{height}^2}$. Why does this formula work?

4. Jacqueline wishes to pack a 3 ½ - foot umbrella in a rectangular suitcase that measures 3 ft x 2 ft x ¾ ft. Is this possible? Why or why not?