Nspire Activity: Have a Heart...Or a Snail!

1. Make sketches for the graphs of the functions listed in #3, 4, and 5 in this activity. These functions are written in the form $y = a + b\cos x$. State the range of each function below each graph. Then, answer the following question.

How do the values of 'a' and 'b' in $y = a + b\cos x$ affect the graph of a trig function?

'b' vertically stretches the graph $y = \cos x$ and represents the amplitude of the function; 'a' translates the graph $y = \cos x$ vertically.

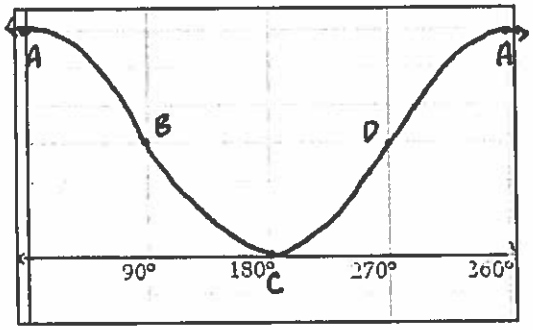
Part 1: Investigating the case of a = b

2. Open the file 'polar3'. Move to page 2.2 of the document. You will see the graph of $y = 3 + 3\cos x$ on the left panel and a polar grid on the right panel.

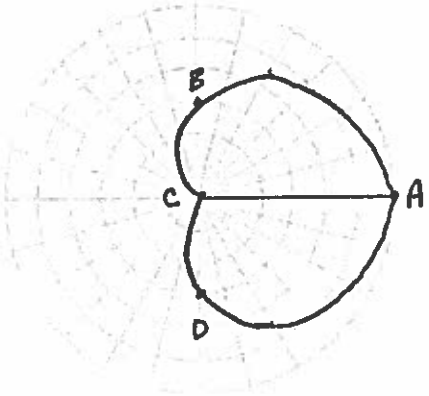
   You will be observing the polar graph formed by tracing the graph of the function $y = 3 + 3\cos x$. As the graph is traced for 'x' values from [0, 360°], corresponding points in terms of 'r' and 'θ' for the graph of the polar equation $r = 3 + 3\cos\theta$ will be plotted on the polar grid.

3. Press play and observe the polar graph formed. Make a sketch of the polar graph and answer the questions that follow.

Function: $y = 3 + 3\cos x$



Polar Equation: $r = 3 + 3\cos\theta$



Range: [0, 6]

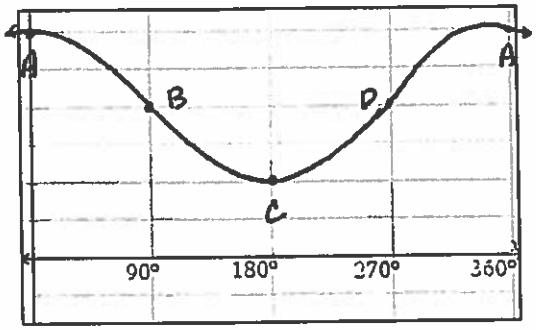
Look at the range listed for the trig function above. Explain connections between the range of the trig function and the span of 'r' values shown in the polar graph that support its shape.

'r' ranges from 6 initially (at $\theta = 0°$) and slowly decreases until $r = 0$ (at $\theta = 180°$) causing a heart-like shape with a cusp to form; after $\theta = 180°$, r slowly increases forming the other side of the heart-shape.

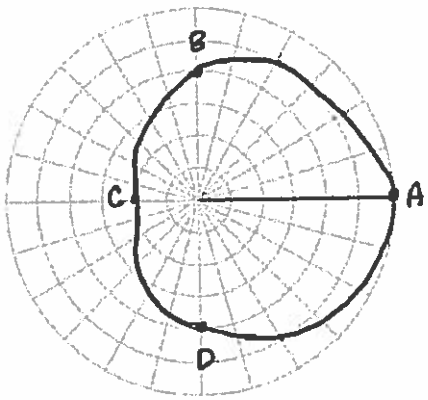
Part 2: Investigating the case of a > b

4. Move to page 3.2 of the document. Press play and observe the polar graph formed by tracing the function $y = 4 + 2\cos x$. Make a sketch of the polar graph and answer the questions that follow.

Function: $y = 4 + 2\cos x$



Polar Equation: $r = 4 + 2\cos\theta$



Range: [2, 6]

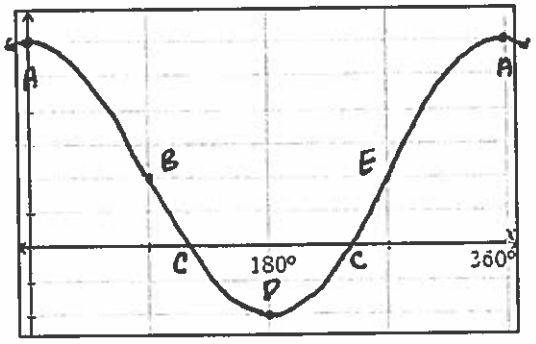
How is this polar graph different than when $r = 3 + 3\cos\theta$? Explain connections between the range of the trig function and the span of 'r' values shown in the polar graph that support its shape.

'r' ranges from 6 initially (at $\theta = 0°$) and slowly decreases until $r = 2$ (at $\theta = 180°$); this causes more of a 'dent' in the curve than a cusp as with $r = 3 + 3\cos\theta$; after $\theta = 180°$, r increases and forms the other side of the shape

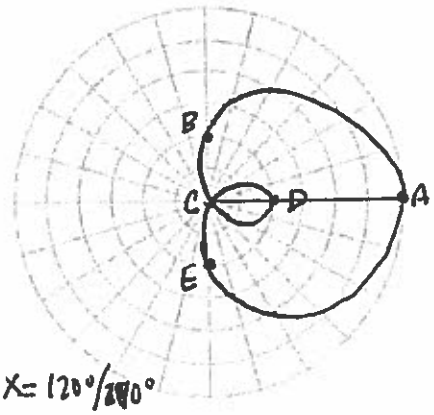
Part 3: Investigating the case of a > b

5. Move to page 4.2 of the document. Press play and observe the polar graph formed by tracing the function $y = 2 + 4\cos x$. Make a sketch of the polar graph and answer the questions that follow.

Function: $y = 2 + 4\cos x$



Polar Equation: $r = 2 + 4\cos\theta$



$2 + 4\cos x = 0$

$\cos x = -\frac{1}{2}$ $x = 120°/240°$

Range: [-2, 6]

How is this polar graph different than when $r = 3 + 3\cos\theta$? Explain connections between the range of the trig function and the span of 'r' values shown in the polar graph that support its shape.

'r' ranges from 6 initially (at $\theta = 0°$) and decreases until $r = 0$ (at $\theta = 120°$) where the graph forms a cusp; then 'r' < 0 and an inner loop forms on the graph while $r < 0$; after $\theta = 240°$ 'r' increases and forms the rest of the shape