In any situation, the outcomes (like the proportions of ingredients in a sample of candy) may vary due to chance. If you toss a coin 1,000 times, you would expect to get somewhere around 500 heads. The chi-square test for goodness-of-fit can be used to determine if there is a significant difference between the proportions you would expect to get in a sample and the proportions you actually get.

Suppose a certain popular brand of candy pieces comes in five colors. A student counted the number of Pieces of each color in a bag and found the results shown in the table to the right.

Color	Observed	Expected	Observed – Expected	$\frac{(O-E)^2}{E}$
Yellow	11			
Red	19			
Blue	25			
Orange	17			
Green	13			
			Sum	

Based on these data, is it likely that this bag of candy came from a manufacturing process that was designed to produce equal proportions of each color? One way to answer this question is to perform a statistical procedure called a chi-square test for goodness-of-fit, checking to see how well the sample distribution fits the theoretical distribution.

A chi-square test requires that the...

- samples are chosen randomly
- samples are independent
- sample size is large enough for the expected values to be at least 5

For this example we will assume that all of these requirements are met.

1. How many pieces of candy were in the bag? If the proportions of each color were the same, how many pieces of each color would you expect to find?

Enter these values in the table above. These values are called the *expected values*. The actual counts of each color of candy in the table are called the *observed values.*

2. Complete the next two columns of the table. Do you think that there is a considerable difference between the observed values and the expected values?

3. This chi-square test involves quantifying the extent of the difference between the observed and expected values for each of the colors. Does it make sense to find the sum of the differences (Observed – Expected) to describe the total difference? Why or why not?

4. For each color, compute "(Observed– Expected)^2 / Expected" and enter this value in the last column. Find the sum of these five values. This is called the chi-square, represented by X^2.

The question now is whether the chi-square found is large or small. To find out, you can compare your observed chi-square to the theoretical distribution of the probabilities that a chi-square would be greater than or equal to any given value. The cumulative chi-square density can be calculated on your handheld (**MENU > Statistics > Distributions > X^2 Cdf**)

The **Lower Bound** is the chi-square value calculated in Exercise 3, above.

The **Upper Bound** is 1,000. The number 1,000 is used as a large number, so you can find the probability that a chi-square would fall between your value and 1,000 by chance.

The **Degrees of Freedom** is 4. Since there are 5 colors, the number of candies possible for the fifth color is determined by the number of candies for each of the other four colors. That is, there are only four independent variables; the last color depends on how many of the other four there are.

The X^2 Cdf command computes the area of the region under the graph of the X^2 density function from the chi-square value to, in this case, 1,000. This area is called the *p*-value, the probability that a chi-square would be as large as or larger than the observed value simply by chance.

5. What is the *p*-value for the bag of candy?

A chi-square of 7.0588 would yield a *p*-value of 0.1328. This value means that if the colors of the candy were in fact distributed equally, a chi-square of 7.0588 or higher would occur by chance about 13% of the time. Something that happens 13% of the time is not considered too unusual, so a statistician would conclude there is not enough evidence to state that the bags of candy did not come from a process that produced equal numbers of the colors. Statisticians usually make the same judgment if the *p*-value is more than 5%. If the *p*-value is less than 5%, a statistician would conclude that there is sufficient evidence to reject the assumption that the bags of candy came from a process that produced equal numbers of the colors.

6. Based on the answer to Exercise 5, is there sufficient evidence to reject the hypothesis that the bags of candy came from a process that produced equal numbers of the colors? Why or why not?

Additional Exercises

A. Another student opened a bag of a different brand of candy, counted the number of pieces of each color and found the results shown in the table below.

Color	Observed Amount	Expected	Observed – Expected	$\frac{(O-E)^2}{E}$
Brown	15			
Yellow	14			
Red	16			
Blue	35			
Orange	29			
Green	24			

Repeat Exercises 1–6 from the activity, making any necessary modifications, and use a chi-square test to determine if it is likely that this bag of candy came from a manufacturing process that was designed to produce equal numbers of each color.

B. A third student looked at the manufacturer's Web site for this second brand of candy and saw the claim that the candy is made in the proportions: Brown 13%, Yellow 14%, Red 13%, Blue 24%, Orange 20%, and Green 16%. Because of the distribution of colors in this bag, she was suspicious of the accuracy of the manufacturer's claim. Repeat Exercises 1–6 from the activity and use the chi-square test to determine if it is likely that this bag of candy came from a manufacturing process that was designed to produce these proportions of each color.

Color	Observed Amount	Expected	Observed – Expected	$\frac{(O-E)^2}{E}$
Brown	15			
Yellow	14			
Red	16			
Blue	35			
Orange	29			
Green	24			

