Mathematical Methos CAS Unit 1 Investigative Project
Matrices and Simultaneous Equations

SINGULAR MATRICES.

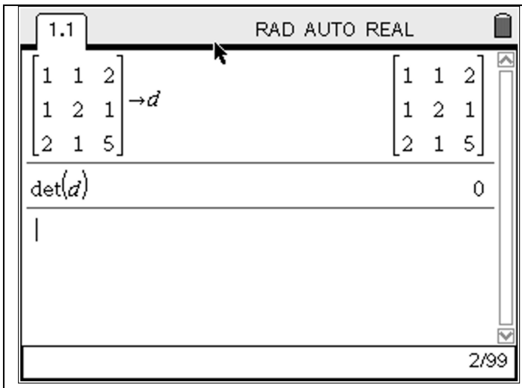
When a determinant of a matrix is equal to zero, such a matrix is called a singular matrix.

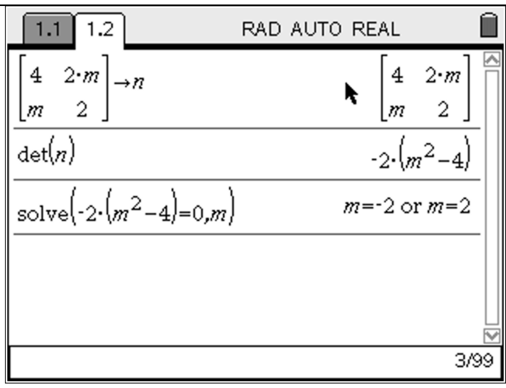
1. Determine which of the following are singular matrices.

$$A=\begin{bmatrix}1 & 3\\ -2 & 5\end{bmatrix}\quad B=\begin{bmatrix}4 & 2\\ 10 & 5\end{bmatrix}\quad C=\begin{bmatrix}3 & 5\\ 0 & 1\end{bmatrix}\quad D=\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 1\\ 2 & 1 & 5\end{bmatrix}\quad E=\begin{bmatrix}1 & 0 & -2\\ 3 & 0 & -1\\ 2 & 1 & 4\end{bmatrix}$$

2. Find the values of a for which matrix $M=\begin{bmatrix}a & 2\\ 3 & 1\end{bmatrix}$ is singular.
3. Find the values of m for which matrix $N=\begin{bmatrix}4 & 2m\\ m & 2\end{bmatrix}$ is singular.
4. Find the values of k for which $P=\begin{bmatrix}k & 2\\ 4 & k\end{bmatrix}$ is singular.
5. Can we find an inverse of matrix B? Justify your answer.

CAS TIP:



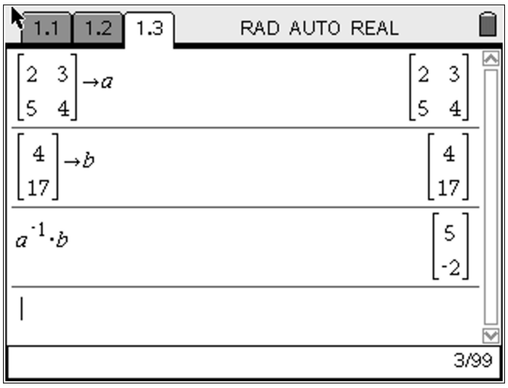


MATRIX EQUATIONS.

To solve a matrix equation we need to find an inverse matrix. For example to solve an equation $AX=B$, where $A=\begin{bmatrix}2 & 3\\ 5 & 4\end{bmatrix}$ and $B=\begin{bmatrix}4\\ 17\end{bmatrix}$ we need to follow these steps:

Pre-multiply both sides by A^{-1}.
$A^{-1}AX=A^{-1}B$
We know that $A^{-1}A=I$ and therefore the equation simplifies to
$X=A^{-1}B$

Now we can use the calculator to solve the equation as follows: store both matrices as A and B. Then find $A^{-1}B$



So matrix $X = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$.

6. Solve the following matrix equations:

a. $CY = D$, where $C = \begin{bmatrix} 3 & 5 \\ 2 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 4 \\ 11 \end{bmatrix}$.

b. $MX = N$, where $M = \begin{bmatrix} 3 & 2 & -1 \\ 1 & -1 & 2 \\ 2 & 3 & -1 \end{bmatrix}$ and $N = \begin{bmatrix} 14 \\ -8 \\ 13 \end{bmatrix}$.

SOLVING SIMULTANEOUS EQUATIONS USING MATRICES.

A pair of linear simultaneous equations

$$\begin{cases} 2x + y = 4 \\ 3x + 4y = -1 \end{cases}$$

can be written in matrix form as follows:

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

and then solved using the method as in the previous section.

$$AX = B$$
$$X = A^{-1}B$$

7. Solve the following simultaneous equations using matrix method.

a. $\begin{cases} 2x + y = 4 \\ 3x + 4y = -1 \end{cases}$

b. $\begin{cases} 7x + 11y = 18 \\ 11x - 7y = -11 \end{cases}$

The pair of linear simultaneous equations can be interpreted geometrically as two straight lines. There are three possibilities with two intersecting lines.

Case 1: Two lines intersect in a point. This is called a **unique** solution.
Case 2: Two lines are parallel. There are no solutions.
Case 3: Two lines coincide. There is an infinite number of solutions.

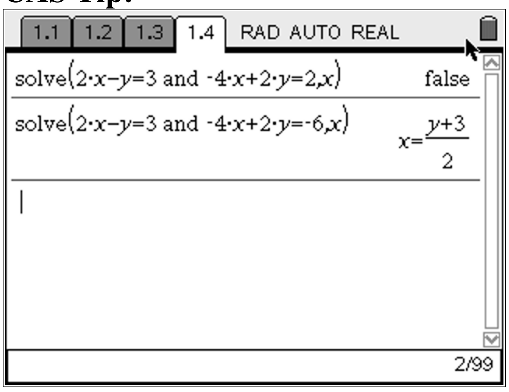
8. a. Write the system of equations $\begin{cases} 2x - y = 3 \\ -4x + 2y = 2 \end{cases}$ in matrix form $AX = B$.

b. Find the determinant of matrix A.
c. Can we find the inverse matrix A^{-1}?
d. Is there a unique solution to the above system of equations?
e. Use your calculator to draw both lines on one set of axes. What do you notice?
f. Use solve feature on your calculator to solve the above system of equations. Interpret the answer.

9. a. Write the following system of equations $\begin{cases} 2x - y = 3 \\ -4x + 2y = -6 \end{cases}$ in matrix form $AX = B$.

b. Find the determinant of matrix A.
c. Can we find the inverse matrix A^{-1}?
d. Is there a unique solution to the above system of equations?
e. Use your calculator to draw both lines on one set of axes. What do you notice?
f. Use solve feature on your calculator to solve the above system of equations. Interpret the answer.

CAS Tip:



It follows from the above investigation that:

A unique solution exists when $\det(A) \neq 0$. **The inverse matrix** A^{-1} **can be evaluated and the solution can be found. Two lines intersect in a point.**

When $\det(A) = 0$, **there is no unique solution. Two lines are either parallel which means there are no solutions or two lines coincide, which means there is an infinite number of solutions.**

10. Consider the system $\begin{cases} 2x - 3y = 8 \\ 4x - y = 11 \end{cases}$

a. Write the equations in the form $AX = B$ and find det(A).
b. Does the system have a unique solution? If so, find it.

11. Consider the system $\begin{cases} 2x + ky = 8 \\ 4x - y = 11 \end{cases}$

a. Write the system in the form $AX = B$ and find det(A).

b. For what value of k does the system have a unique solution? Find the unique solution in terms of k.

c. Find k when the system does not have a unique solution. How many solutions does it have in this case?