EXPLORATIONS

Activity 6

It's a Small World

The populations of the United States and the world have grown rapidly during recent history. Many different factors can affect the rate at which a population changes, including the climate, technology, and the economy. It is important in a number of areas to be able to predict future populations. Different mathematical models are appropriate over different lengths of time.

Introduction

In this activity, you will collect population data for the world on ten different days. You will then find a model for this data set assuming that over a short period of time the data can be considered to be linear. Finally, the model will be evaluated over a much longer period to determine the validity of the linear model.

Equipment Required

- Computer
- TI InterActive!™ software
- A working Internet connection
- Adobe® Acrobat® Reader software

Collecting the Data

1. Start TI InterActive! The software opens to a new, blank document.
2. Title your document *Small World*, and add your name and the date. Click the Save button  to save and name your document.
3. Click the Web Browser  to open the TI InterActive! browser. Click on the Data Sites button  . Under the **Activity Book Links** category, click on *TI InterActive! Data Collection and Analysis*. Choose **Activity 6: It's a Small World**.
4. Once the page has been loaded in the browser, click **Population Clocks**. On the POPClocks page you should see the current estimated population of the United States and of the world.

5. Record the date, day number, and population of the world in the table below. Use the number of days since the year 2000 began in column two, allowing January 1, 2000 to be day one. You will need ten different days on which to collect this data. Although taking readings at the same time each day might be the best technique, differences of a few hours either way will not have a significant effect on the model.

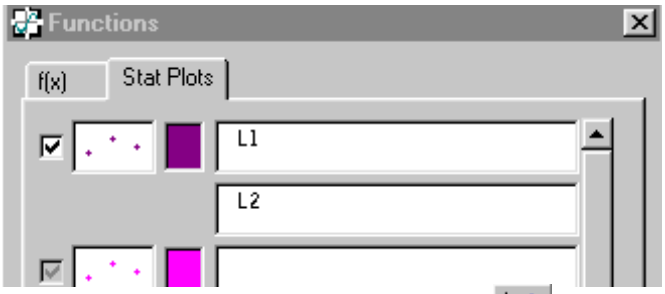
Date	Day Number	Population of the World

Recording the Data

1. Click the List button  , then click the empty cell at the top of list **L1**. Type the initial value and then press the down arrow key on the keyboard to move to the next cell. Continue entering the number of days since January 1, 2000 until you have entered all of the day values into **L1**.

2. Click the empty cell at the top of list **L2**. Enter the corresponding population values for each day that you recorded in **L1**. You may need to resize **L2** so you can read the data. To do this, click and drag the bar on the right side of the cell labeled **L2**.

3. Click the Scatter Plot button  and then click the Stat Plots tab. In the uppermost text box, type **L1** to specify it as the list containing the *x* coordinates. Press the Tab key and move to the second text box. Type **L2** to specify the list containing the *y* coordinates.



4. Press Enter, and then click the Zoom Statistics button  . The viewing boundaries adjust automatically to show all the plotted data.

5. The plot of number of days versus the population of the world should appear to be linear in nature. Click on the Save to Document button  to copy this plot into your TI InterActive!™ document.

Analysis and Questions

1. Click on Trace and move along the plot of your data using the arrow keys. Choose two points that would appear to lie on a line of best fit and make a note of the day numbers of these points below.

   Day # Day #

2. Record the day # and population for the two days you chose above as ordered pairs in the space below. Refer to the table or Data Editor for the exact values of the coordinates.

3. Click on Save to Document  to return to your TI InterActive! document. Click on Math Box  and use the two points you selected above and the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the slope of the line of best fit. Record your answer below.

4. A simple way to describe slope is change in y over change in x. What is the real-world interpretation of the slope that you found in the question above?

5. To find the y-intercept of your linear model, you will need to solve the equation $y = mx + b$ for b. To do this, click on Math Box  and enter **solve(y=mx+b,b)** substituting the value of the slope you found in step **3** for m and either of the points you selected in step **2** for x and y. Press Enter to solve for b, and record the solution below.

   $b =$

6. A simple way to describe the y-intercept is to say it is the point where the graph crosses the y-axis. In this model, what is the real-world meaning of the y-intercept?

7. Use the values of *m* and *b* that you found to write a linear model for your data in the form $y = mx + b$.

8. Double-click on the saved graph in your document to return to the Graph window. Click on the f(x) tab of the Functions dialog box and enter the equation you found in step **7**. Press Enter to turn on the function and graph it. How well does it fit the data you collected?

9. You can use the equation you just found to predict future or estimate past population values for the world. To simplify this, you can define a function with the values you found above. Click on Save to Document  to return to your TI InterActive! document. Click on Math Box  and type **pop(x):=mx+b** using the values you found for *m* and *b* above and press Enter. You can now find any population value for day *x* by typing **pop(x)** in a Math Box and pressing Enter. To start, predict the population of the world one year from today by adding 365 to today's day number. (For example, if today was the tenth day, **pop(375)** would give the population one year from today.) Record this value in the space provided.

10. How reliable do you feel that your prediction above is? Discuss your reasoning.

11. The United States Census bureau originally predicted that the world would attain a population of six billion people on July 19, 1999. Use the Math Box and **pop(x)** to find the number of people your model estimates that there were on that date and record your answer on the space provided. (An easy way to determine the day number is to find what day of the year July 19, 1999 was and add that to –365.)

12. Did your model produce a reasonable value for this date? Explain.

13. On January 1, 2000 over six billion people celebrated the new year. According to your model, how many people will celebrate the new year on January 1, 3000? Use the Math Box and **pop(x)** to find this value and record it below.

14. Does the answer you got in the problem above seem reasonable? Discuss why or why not.

15. Click on the Math Box. Use TI InterActive!™ to determine when your model predicts that the population of the world was zero. To do this, type **solve(0=mx+b,x)** substituting your values for *m* and *b* and press Enter. Remember to convert your answer from days to years. Is this answer reasonable? Explain.

16. Based on the questions above, when do you think a linear model would be appropriate to use in predicting population values? When would it be inappropriate? Explain your answer.

17. Save and print your TI InterActive! document.

Extensions

- Repeat this activity using the population of the United States rather than the world.
- Choose a date during the last week of the school year and use your model to predict the population of the world on that day. When the day you chose arrives, check the population clock and compare your prediction to the actual value.
- Collect population values from over the course of history and set up a plot of the data. Try to find an appropriate mathematical model for population growth over long periods.

Teacher Notes

Activity 6: It's a Small World

Math Concepts

- Internet Data Collection
- Linear Function

Activity Notes

- Begin collecting data a couple of weeks in advance of the date you plan to do this activity. Daily commitment is only a couple of minutes and will allow different students to experience using the web.
- Students need to be careful when determining the day number. Remind them to take into account the different lengths of various months.

Sample Data

Date	Day Number	World Pop.
1/06/00	6	6,036,284,662
1/07/00	7	6,036,524,192
1/08/00	8	6,036,703,934
1/09/00	9	6,036,955,290
1/10/00	10	6,037,177,346
1/11/00	11	6,037,366,570
1/12/00	12	6,037,547,288
1/13/00	13	6,037,736,380
1/14/00	14	6,037,964,912
1/15/00	15	6,038,165,017

Analysis and Questions - Key

1. Day #7, day #14
2. (7, 6036524192) (14, 6037964912)
3. The slope is approximately 205817.
4. The slope represents the increase in world population per day.
5. The value of *b* is 6035083474.
6. The *y*-intercept represents the population of the world at the beginning of the year 2000.
7. $y = 205817x + 6035083474$
8. The model fits the data very well.
9. The world population one year from day 375 will be 6,112,267,849.

10. The prediction should be close, as there should not be any major changes in the rate at which the population is growing over the next year.

11. July 19th is the 200th day of the year. With respect to Jan 1, 2000, this would be day -165. The population on day -165 was 6,001,126,669.

12. The value is reasonable. The percentage error is very small.

13. There are approximately 365,000 days in a millennium. (**Note**: Some students may wish to be more precise here.) The predicted population is 81,158,291,474.

14. The answer is too large. At some point, population growth will begin to level off due to food supplies and overcrowding.

15. The model says the population of the world was zero on day -29,333 or 80 years ago. This is obviously not true; there were many people in 1920.

16. A linear model is only appropriate for short-term population growth. Different models such as exponential or logistic would be better for long term models.