Another Piece of Pi Monte

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Teachers Notes & Answers

7 8 **9** **10** 11 12

TI-Nspire | Investigation | Student | 45 min

Introduction

The Monte-Carlo technique uses probability to model or forecast scenarios. In this activity the Monte Carlo technique will be used to estimate an area; however the learning focus is on the logic and understanding rather than the single numerical result. The numerical result simply verifies that the processes work.

Creating the Simulation

Start a **new document** and insert a **Graph application**.

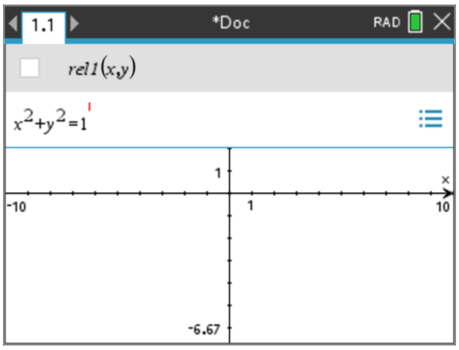
Use the **[Menu]** to change the graph type:

Graph Entry/Edit > **Relation**

Graph the relation:

$$x^2 + y^2 = 1$$

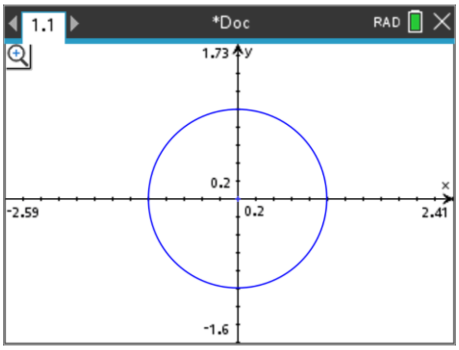
Type in the equation then press **[Enter]** to graph it.



The graph is the focus so it is useful to zoom in a little so that it fills the screen a little more.

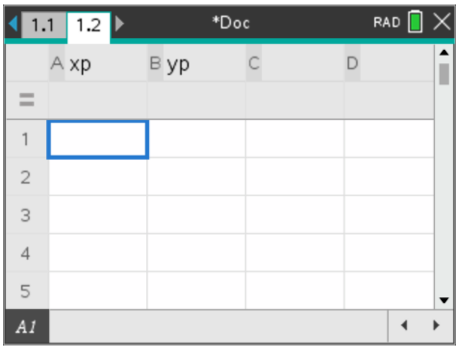
[Menu] > Window/Zoom > Zoom In

The mouse now appears as a magnifying glass. Place it as close as possible to the origin and click. Click again. Three clicks will zoom too much. If you inadvertently zoom too much you can select zoom out or press **Ctrl** + **Z** (undo).



The next task is to create two lists. A spreadsheet is a great place to work with lists. Press **Ctrl** + **I** and insert a Spreadsheet application.

Navigate to the top of the spreadsheet (shown opposite) and type in the names for the lists: **xp** and **yp**.



Author: Peter Fox

The TI-Nspire CX II contains lots of keyboard short cuts. Ctrl + I = Insert; Ctrl + C = Copy. You may be able to guess a lot more. Check out our Short Cuts documents and video tutorials on YouTube.

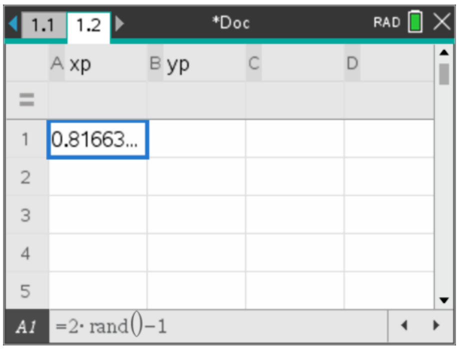
Keys can be pressed successively rather than simultaneously to access short cuts.

Navigate to cell A1. Cells are referenced the same as any other spreadsheet application, similarly formulas start with an 'equals' sign.

In cell A1 type the formula:

= 2 rand() – 1

A random number will be generated[1], notice that the formula used to generate the random number is visible at the bottom of the screen. Press Ctrl + R and notice what happens to the random number.

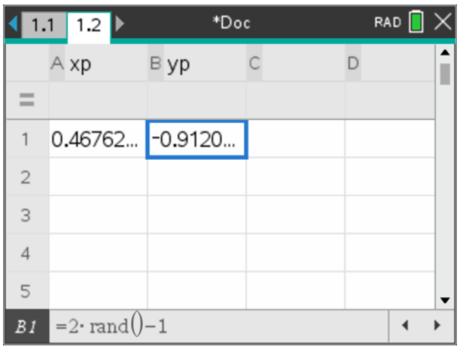


Question: 1.

Rand() generates random numbers between 0 and 1. What sort of numbers does: 2rand() – 1 generate?
Answer: The random numbers will vary between -1 and 1.

The formula also needs to be entered into cell B1. This can be done by retyping the expression, preceded again by the '=' sign.

Alternatively, while cell A1 is highlighted press Ctrl + C (Copy), navigate to cell B1 and press Ctrl + V. (Paste).



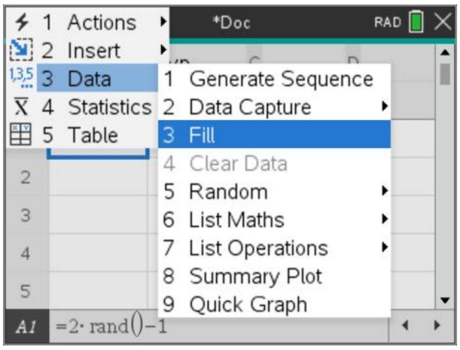
The same formula needs to be entered in all the cells: A1 to A25. Rather than typing or copying a quicker approach is available.

Navigate to cell A1. With cell A1 highlighted press:

[Menu] > **Data** > **Fill**

Arrow down to cell A25 (notice that a dotted line flows down) … press **[Enter]** to generate 25 random numbers.

Repeat this process for the second column: B1 to B25.



[1] **Random Numbers**: Electronic devices generate pseudo random numbers. If your 'random' number is the same as somone else's you should change your 'seed' value. In a Calculator application type: randseed #### and enter a four digit (#) number.

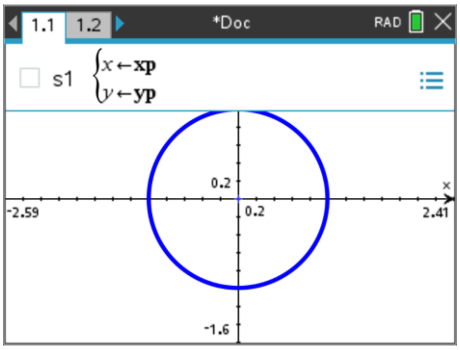
The random points need to be plotted in the Graph application. Navigate back to the Graph application by pressing Ctrl + Left (Navigation pad) or by clicking on the page tab.

Use the menu to change the graph type to a Scatter Plot.

[Menu] > **Graph Entry/Edit** > **Scatterplot**

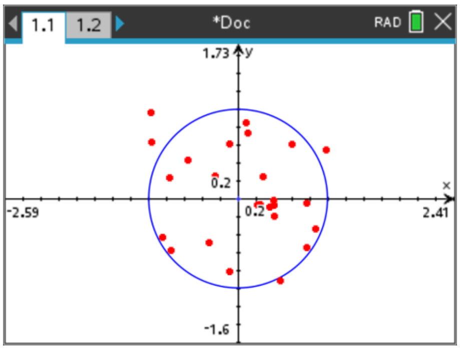
S1 = Scatterplot One

Plot XP on the x axis and YP on the y axis.



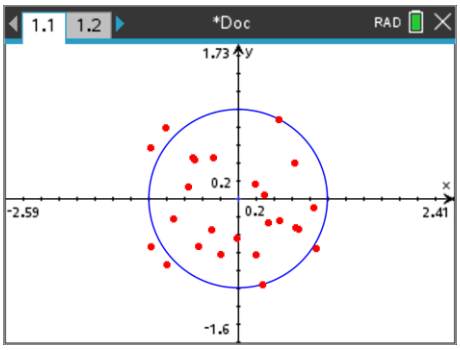
The 25 points plotted. We are interested in the proportion of points that land inside the circle. In the image shown opposite there are 4 points that fall outside the circle, therefore 21 points inside. The proportion of points inside the circle is therefore:

$$\frac{21}{25} = 0.84$$



To generate another sample, return to the spreadsheet (Ctrl + Right) and press Ctrl + R to generate another 25 random points. Navigate back to the graph and determine the proportion of these new points that fall inside the circle.

In the screen shown opposite there are 5 points that are clearly outside the circle, there are two others that appear to be right on the circumference. You could zoom in on these points, however, their precise location can be checked using a simple formula.

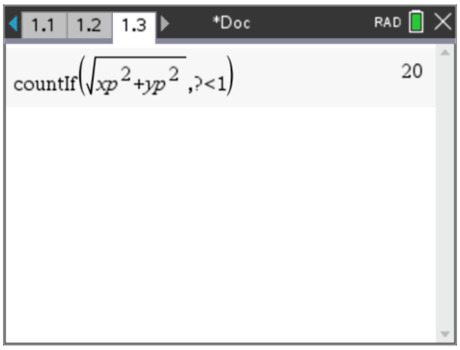


Insert a Calculator application and enter the following formula:

$$countif\left(\sqrt{xp^2 + yp^2}, ? < 1\right)$$

The " countif " command can be typed directly or selected from the catalogue. The question mark (?) can be accessed from the punctuation key or symbols template.

In this simulation the " countif " command identified 20 points that lie inside the circle.



Question: 2.

What is the mathematical expression: $\sqrt{xp^2 + yp^2}$ doing?

Answer: It is using Pythagora's theorem to determine the distance from the origin to the each randomly generated point.

Question: 3.

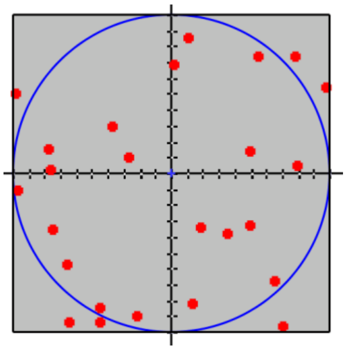
Generate 10 samples each of size 25 and record the proportion of points that land inside the circle in each trail.
Answer: Answers will vary, the typical proportions are close to 0.8.

Trial:	1	2	3	4	5	6	7	8	9	10
Proportion:										

Question: 4.

Based on your data so far, estimate the proportion of points that land inside the circle.

Answer: Approximately 80% of points will land inside the circle.



The randomly generated points all fit inside a square. The proportion of points landing inside the circle represents the proportion of the square's area occupied by the circle.

Question: 5.

Given the circle has a radius of 1 unit:

i) Determine the area of the square.

Answer: Width = 2 ∴ Area = 4units^2

ii) Use the proportion determined in Question 4 to provide an estimate for the area of the circle.

Answer: Proportion $\approx 0.8 \times 4 = 3.2$

iii) Explain how this process can be used to estimate a value for π.

Answer: Area of a circle is $A = \pi r^2$, since r = 1, then the process generates an approximate value for π.

iv) Suggest and explore ways to improve the estimate.

Answer: A larger sample will produce a better estimate. For senior mathematics classes, students should consider what a sampling distribution suggests with regards to 'larger' sample vs 'more' samples.

Extension:

Write a program to automatically run the simulation with larger and large sample sizes.

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