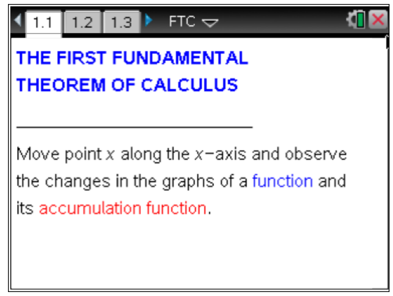
The First Fundamental Theorem of Calculus

Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *FTC.tns*.



The accumulation function, $A(x)$, measures the definite integral of a function f from a fixed point a to a variable point x. In this activity, you will explore the relationship between a function, its accumulation function, and the derivative of the accumulation function. These observations will help you better understand the first Fundamental Theorem of Calculus.

Move to page 1.2.

1. The graph shown is of the function $y = f(x)$. The **accumulation function** of $f(t)$ from a to x is given by $A(x) = \int_a^x f(x)dx$. The accumulation function measures the definite integral of f from a to x. For example, if you set a to –3, $A(2) = \int_{-3}^{2} f(x)dx$, you get the value of the definite integral of f from –3 to 2.

   Drag the point x along the x-axis to determine the values of the accumulation functions below:

   a. $A(3) = \int_{-3}^{3} f(x)dx =$ __________________

   b. $A(0) = \int_{-3}^{0} f(x)dx =$ __________________

   c. $A(-1) = \int_{\square}^{\square} f(x)dx =$ __________________

Move to page 1.3.

2. The top graph shows the original function, $y = f(x)$, and the shaded region between the graph of the function and the x-axis as the point x is dragged along the x-axis. The bottom graph shows the value of the definite integral for each upper limit x, with lower limit a = –3. Drag point x along the x-axis in the top graph to observe the relationship between the two graphs.

   a. At what value(s) of x does the accumulation function, $A(x)$, have a local maximum? A local minimum? Explain how you know.

   b. Drag point x to the x-value at which $A(x)$ has a local maximum. What do you notice about the value of the original function, $f(x)$, at that point?

c. Drag point x to the x-value at which $A(x)$ has a local minimum. What do you notice about the value of the original function, $f(x)$, at that point?

d. At what value of x does the accumulation function, $A(x)$, have an inflection point? Explain how you know.

e. Drag point x to the inflection point of $A(x)$. What do you observe about the original function, $f(x)$, at that point?

3. a. Over what interval(s) is $A(x)$ increasing? Decreasing?

b. What do you observe about $f(x)$ over the interval(s) where $A(x)$ is increasing? Over the interval(s) where $A(x)$ is decreasing?

4. Based on your observations in questions 2 and 3, what do you believe the relationship between the functions $f(x)$ and $A(x)$ to be? Explain your reasoning.

Move to page 1.4.

5. The top graph on page 1.4 is the graph of the accumulation function, $A(x)$, for the function $f(x)$ from previous pages.

a. Drag point x and observe the changes in both graphs. What is the graph on the bottom of the page measuring? How do you know?

b. What is the relationship between the bottom graph on page 1.4 and the original function, $f(x)$?

c. Based on your observations, what is the relationship between the functions $f(x)$ and $A(x)$? How do you know? How does this compare to your answer to question 4?

6. Complete the following: $\frac{d}{dx}A(x) = \frac{d}{dx}\int_a^x f(t)dt$ = ____________. Explain your reasoning.

