

## Optimization – ID: 8996

By Steve Ouellette

Time required

45 minutes

## Activity Overview

Students will work through two different optimization problems, one that requires finding an absolute maximum value and one that requires finding an absolute minimum value. With both problems, students will use the Graphs & Geometry application and the manual data capture feature to find these values experimentally. Students will then use the information given in each problem to write an equation in two variables that can be analyzed using differential calculus, yielding the same absolute extreme values that were derived experimentally.

## Concepts

- Optimization
- Differentiation
- Absolute maximum and minimum values

## Teacher Preparation

This investigation offers an opportunity for students to apply their knowledge of finding absolute maximum and minimum values to problems involving optimization. This activity has been designed such that students will start by finding the answers to both optimization problems using experimental methods. This will help students realize that there are often domain restrictions on applied problems and, therefore, methods used to find absolute extreme values should include these restrictions. The graphs produced in this activity will help illustrate the appropriateness of finding an absolute maximum rather than an absolute minimum or vice versa.

- Students should already be familiar with finding absolute extrema using methods of calculus. Specifically they should realize that, on a closed interval, absolute extreme values can occur at the endpoints of the interval or at places where the derivative is equal to zero or undefined.
- The screenshots on pages 2–4 demonstrate expected student results. Refer to the screenshots on page 5 for a preview of the student TI-Nspire document (.tns file).
- **To download the student .tns file and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “8996” in the quick search box.**

## Classroom Management

- This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.
- Students will manipulate pre-made sketches, rather than create their own constructions. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.
- The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the activity is completed successfully.

## TI-Nspire™ Applications

Graphs & Geometry, Lists & Spreadsheet, Notes

One focus question defines this activity: *How can calculus and the concept of absolute maximum and minimum values be used to solve the problem posed in page 1.2?*

Students should already have some experience using derivatives to find absolute maximum and minimum values of a variety of functions. Discuss with students that other constraints must be considered when dealing with application problems, particularly those that affect the domain.

While discussing the problem posed on page 1.2, some students may guess that a rectangle with a fixed perimeter always has a maximum area when it is a square. Point out that this problem is different because the fence will only run along three sides of the rectangle. You may also ask students if they think it makes sense to find an absolute minimum value rather than an absolute maximum value.

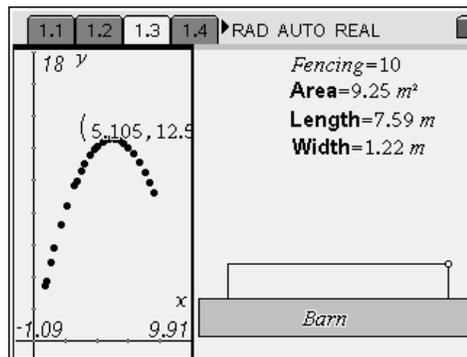
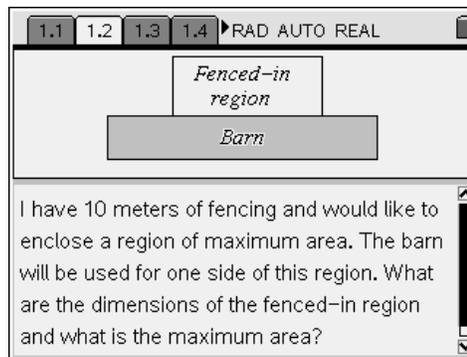
After this initial discussion, instruct students to advance to page 1.3 where they will identify the solution experimentally and then use methods of calculus to find the solution analytically.

### Problem 1 – Maximizing the area of a fenced-in region

**Step 1:** Working cooperatively, students will slowly drag the rectangle to the right, which will increase the length and decrease the width while maintaining a fixed fence length of 10 meters. Remind students to periodically press  $\text{ctrl} + \text{click}$  to manually capture the length and area of the rectangle.

Students will observe that the length that produces a maximum area appears to be near 5. The screen at right shows that the trace command can be used to estimate the vertex of the graph with more precision.

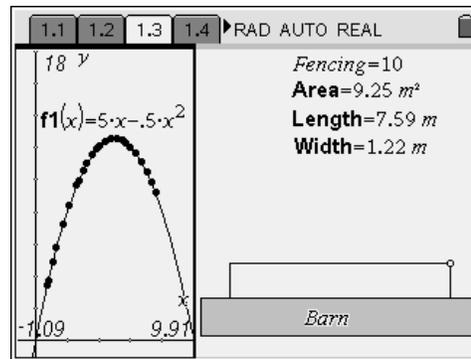
The vertex is exactly located at (5, 12.5), meaning that a rectangle with length 5 m and width 2.5 m will produce the greatest area,  $12.5 \text{ m}^2$ .



**Step 2:** In addition to estimating the vertex, students are asked to find the domain and comment on the shape of the scatter plot. The domain is (0, 10) and the shape of the scatter plot is a parabola.

**Step 3:** The area of the rectangle is given by  $A = lw$ . Also, the equation for the amount of fence is given by  $10 = l + 2w$  or  $w = 5 - \frac{1}{2}l$ . Using substitution, we get  $A = l(5 - \frac{1}{2}l) = 5l - \frac{1}{2}l^2$ .

The screen at right shows the graph of  $A = 5x - \frac{1}{2}x^2$  which confirms that the area function is correct.



**Step 4:** Taking derivatives shows that  $l = 5$  is a critical value, which corresponds to the vertex of the parabola.

$$\begin{aligned} \frac{dA}{dl} &= 5 - l \\ 0 &= 5 - l \\ l &= 5 \end{aligned}$$

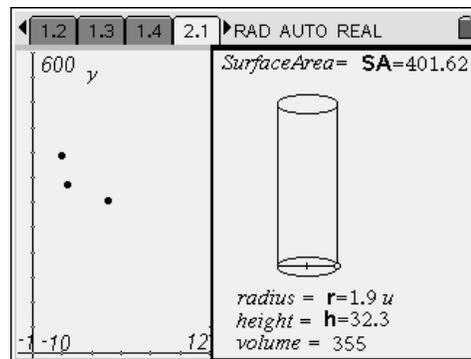
Students should also understand that this value of  $l$  corresponds to the absolute maximum since a parabola only has one critical value. Knowing  $l$  is 5 m, students can find that  $w = 5 - \frac{1}{2}(5) = 2.5$  m and the maximum area is  $A = 5 \cdot 2.5 = 12.5$  m<sup>2</sup>.

Instruct students to advance to Problem 2.

**Problem 2 – Minimizing the surface area of a cylinder**

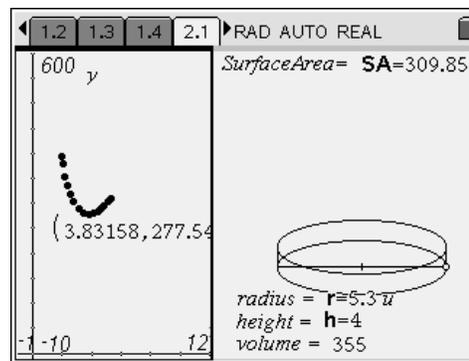
**Step 1:** Students will advance to page 2.1 where they will investigate finding the minimum surface area for a cylinder with a fixed volume.

Students are asked to comment on why minimizing surface area may be a desirable goal. Some students may note that, in manufacturing, minimizing the surface area will help minimize the cost of manufacturing a can. You may want to mention that 355 cm<sup>3</sup> is the volume of a standard soda can.



**Step 2:** In this problem, students will change the dimensions of the cylinder (by dragging the point with the open circle) and observe the radius that produces the minimum surface area. Remind students to periodically press **ctrl** + **⏏** to manually capture the radius and surface area of the cylinder and graph this data as a scatter plot.

**Step 3:** The screen at right shows that the trace command can be used to estimate the minimum value of the radius versus surface area graph. The actual minimum value is located at (3.84, 277.55), meaning that a cylinder with radius 3.84 cm and height 7.67 m will produce the minimal surface area, 277.55 cm<sup>2</sup>.



**Step 4:** In addition to estimating the absolute minimum value, students are asked to find the domain and comment on the shape of the scatter plot. The domain is  $(0, \infty)$  and the shape is given by a rational function. Since the domain is given on an open interval, the only critical point is given by the value of  $r$  for which the derivative of the surface area function is equal to zero.

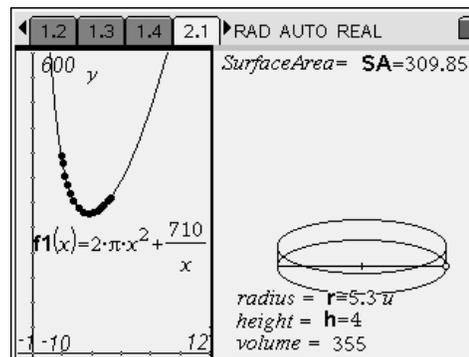
**Step 5:** The surface area function is given by  $SA = 2\pi r^2 + 2\pi rh$ . The fixed volume of 355 ml gives the equation

$$355 = \pi r^2 h \text{ or } h = \frac{355}{\pi r^2}. \text{ Substituting this}$$

second form into the surface area equation

$$\text{gives } SA = 2\pi r^2 + 2\pi r \left( \frac{355}{\pi r^2} \right) = 2\pi r^2 + \frac{710}{r}.$$

This surface area function is shown graphed on the screen at right (with  $x$  used for the radius,  $r$ ).



Now differentiate the surface area function and set it equal to zero:

$$SA' = 4\pi r - \frac{710}{r^2}$$

$$\frac{710}{r^2} = 4\pi r$$

$$r \approx 3.84 \text{ cm}$$

Use the relationship  $h = \frac{355}{\pi r^2}$  to find  $h \approx 7.67 \text{ cm}$ .

The minimum surface area is therefore  $SA \approx 2\pi(3.84)^2 + 2\pi(3.84)(7.67) \approx 277.7 \text{ cm}^2$ .

In general, the surface area of a cylinder with fixed volume is such that the radius is half the height.

### Optimization – ID: 8996

(Student)TI-Nspire File: *CalcAct06\_Optimization\_EN.tns*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

**OPTIMIZATION**

---

**Calculus**

Applications of Absolute Maximum  
and Minimum Values

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

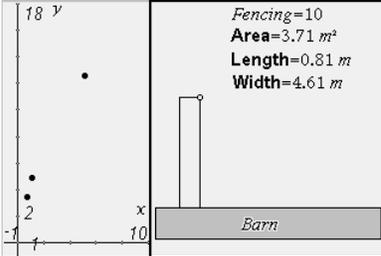
Fenced-in  
region



Barn

I have 10 meters of fencing and would like to enclose a region of maximum area. The barn will be used for one side of this region. What are the dimensions of the fenced-in region and what is the maximum area?

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL



Fencing=10  
**Area=3.71 m<sup>2</sup>**  
**Length=0.81 m**  
**Width=4.61 m**

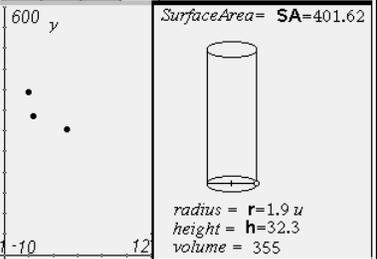
Barn

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

A	xlist	B	ylist	C	D
◆	= capture(lengt = capture(area,				
1	.745001		3.4568		
2	1.085		4.84996		
3	5.185		12.5477		
4					
5					

A1 |

1.2 1.3 1.4 2.1 ▶RAD AUTO REAL



SurfaceArea = SA=401.62

radius = r=1.9 u  
height = h=32.3  
volume = 355

1.3 1.4 2.1 2.2 ▶RAD AUTO REAL

A	radius	B	height	C	surface_area
◆	= capture('r = capture('h = capture(sa,0)				
1	1.91579		30.7881		393.665
2	2.37193		20.0851		334.684
3	5.15439		4.25329		304.677
4					
5					

A1 | =1.91578947367