Activity
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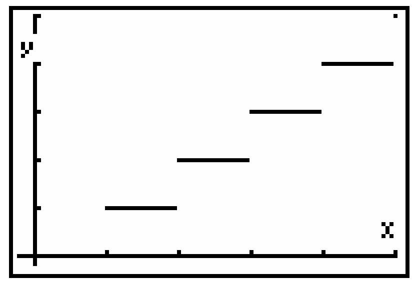
Stepping to the Greatest Integer: The Greatest Integer Function

Not all mathematical functions have smooth, continuous graphs. In fact, some of the most interesting functions contain jumps and gaps. One such function is called the *greatest integer function*, written as $y = \text{int } x$. It is defined as the greatest integer of x equals the greatest integer less than or equal to x. For example, int 4.2 = 4 and int 4 = 4, while int 3.99999 = 3.

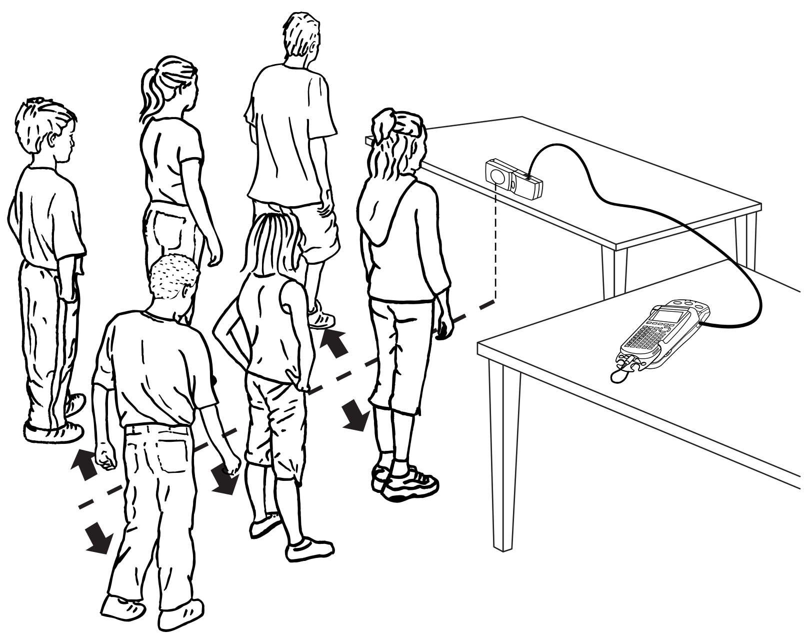
The graph of $y = \text{int } x$ yields a series of steps and jumps as shown here.



In this activity, you will create a function similar to the greatest integer function graph by having a group of students stand in a line in front of a Motion Detector and then step aside one by one. The equation for this graph, in the general form, is

$$y = A \text{ int } (Bx) + C$$

You can find appropriate values for the parameters A, B, and C so that the model fits the data.



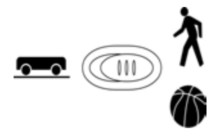
OBJECTIVES

- Use a Motion Detector to collect position data showing evenly-spaced jumps in value.
- Model the position data using the greatest integer function.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
CBR 2 or Go! Motion and direct calculator cable
or Motion Detector and data-collection interface

PROCEDURE



1. Set up the Motion Detector and calculator.
   a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
   b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)

2. Position the Motion Detector as shown in the drawing.

3. Set up EasyData for data collection.
   a. Start the EasyData application, if it is not already running.
   b. Select **File** from the Main screen, and then select **New** to reset the application.
   c. Select **Setup** from the Main screen, then select **Time Graph…**
   d. Select **Edit** on the Time Graph Settings screen.
   e. Enter **0.1** as the time between samples.
   f. Select **Next**.
   g. Enter **100** as the number of samples and select **Next**.
   h. Select **OK** to return to the Main screen.

4. To start, line up six students in front of the Motion Detector as shown in the drawing. Be sure that the spacing between the students is uniform (about half a meter) and that the first student in line is no closer than one meter from the detector.

5. Once you have all students standing in line and ready to move, select **Start** to begin data collection. Have the first student wait for about a second before he or she moves aside. Once the detector is activated, students should step aside in evenly spaced time intervals so that the lengths of the segments appearing on the distance versus time plot are uniform.

   To help gauge the pace, you might have one student tell the others when to step aside by counting out loud every one or two seconds. Be patient; it may take several trials to obtain a plot that resembles a greatest integer function. Data collection will run for ten seconds.

6. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The distance versus time graph should show a series of increasing steps. Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select **Main** and repeat Step 6–7.

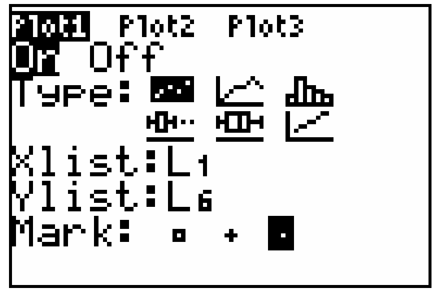
7. Once you are satisfied with the graph, select **Main** to return to the Main screen. Exit EasyData by selecting **Quit** from the Main screen and then selecting **OK**.

ANALYSIS

1. Redisplay the graph using a scatter graph rather than the line graph shown by EasyData. This way the calculator will not incorrectly draw in vertical connecting lines between horizontal steps.

   a. Press [2nd] [STAT PLOT] and press [ENTER] to select Plot 1.

   b. Change the Plot1 settings to match the screen shown here. Press [ENTER] to select any of the settings you change.

   c. Press [ZOOM] and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the *x* and *y* ranges set to fill the screen with data.

   d. Press [TRACE] to determine the coordinates of a point on the graph using the cursor keys.

2. You can model the function with the equation $y = A\ \text{int}(Bx) + C$. Compare the graph with the graph of the greatest integer function on the first page. The graph will be different from the simple graph of $y = \text{int}\ x$ in several ways. The graph will be shifted upward by some amount *C*. The vertical spacing between the steps will not be one meter, but will be closer to a value *A*. (You can think of this parameter as creating a vertical stretch of the graph.) Finally, the graph is stretched horizontally by a parameter *B*.

   You can estimate values of these three parameters from the graph. Trace across the graph to the left-hand edge, at $x = 0$. This will be the vertical offset of the graph, or the parameter *C*. Record this value as *C* in the Data Table on the *Data Collection and Analysis* sheet.

3. Trace to the right across the graph. Estimate the typical magnitude of the vertical spacing between steps. Use several steps to determine the value. Record the value as *A* in the Data Table. You will refine this value shortly.

4. Trace back across the graph, and estimate the typical length in time of the steps. Take the inverse of the time. Record this inverse time value as *B* in the Data Table. You will later refine this value as well.

5. Enter the model equation $y = A\ \text{int}(Bx) + C$.

   a. Press [Y=].

   b. Press [CLEAR] to remove any existing equation.

   c. Enter A*int(B*X) + C. Obtain the int operation by pressing [MATH] and then pressing [▶] to move to the NUM menu. Select int(to paste it to the equation line.

   d. Press [2nd] [QUIT] to return to the home screen.

6. Set values for the parameters *A*, *B*, and *C* and then look at the resulting graph. To obtain a good fit, you will need to try several values for each, but change only one value at a time. Use the steps below to store different values to the parameters. Start with values from the Data Table.

   a. Enter a value for the parameter *A*. Press [STO▶] A [ENTER] to store the value in the variable A.

   b. Repeat for parameters *B* and *C*.

   c. Press [GRAPH] to see the data with the model graph superimposed.

   d. Press [2nd] [QUIT] to return to the home screen.

Experiment until you find values that provide a good fit for the data. Record the optimized values for the parameters in the Data Table.

⇒ Answer Questions 1-7 on the *Data Collection and Analysis* sheet.

EXTENSION

Consider the equation $y = 3 \text{ int } (0.25x) + 5$. Discuss the significance of the numbers in this equation. Write a set of instructions describing the way a group of students would have to move in front of a motion detector to create a data set for which this equation would be an appropriate model. Collect data as before and follow the set of instructions that you developed. Enter the given equation in the Y= list. How well does this equation fit the data you collected?

Name other real world situations that can be modeled using the greatest integer function.

DATA COLLECTION AND ANALYSIS

Name ______________________________

Date ______________________________

DATA TABLE

Parameter	From Graph	After Optimization
A		
B		
C		

QUESTIONS

1. What is the physical significance of the value of C in the model equation $y = A\ \mathrm{int}(Bx) + C$?

2. What is the physical significance of the value of A in the model equation?

3. What is the physical significance of the value of B in the model equation?
   Hint: Consider the value of $1/B$.

4. The calculator is not plotting the greatest integer function quite correctly. Study the graph and describe what the calculator is doing wrong.

5. Suppose a new group of students repeats this activity under the following conditions. The students are farther away at the start, they are spaced closer together, and they step off more quickly. State whether each constant A, B, and C would increase or decrease.

6. The greatest integer function has some interesting business applications. Suppose a phone company charges \$0.25 for the first minute and \$0.15 for each additional minute for a call to a certain exchange. Develop a formula, involving the greatest integer function, to describe the amount charged as a function of the amount of time spent on the phone. Remember, a customer talking for 3.01 or 3.99 minutes must be charged for 4 minutes of conversation. Give a formula with this behavior below.

7. Check to see if the formula developed in the previous question is correct if a person talks for an integer number of minutes. If not, what is wrong with the formula in this case? If necessary, modify it to make it correct in all cases. Explain your method.

