

Conditional Statements

ID: 8744

Time required
40 minutes

Activity Overview

In this activity, students will write logical statements related to the given conditional statement. They will explore whether the statements are true or false and find counterexamples for false statements. These explorations will involve the slopes of parallel and perpendicular lines and lengths of collinear and noncollinear segments.

Topic: Inductive & Deductive Reasoning

- *Write the inverse, converse, and contrapositive statements corresponding to a given conditional statement.*
- *Use a counterexample to prove that a statement is false.*

Teacher Preparation and Notes

- *This activity is designed to be used in a high school or middle school geometry classroom. It is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively.*
- *Before beginning this activity, students should be familiar with the terms inverse, converse, and contrapositive.*
- *Students will discover the following concepts:*
 - *Parallel lines have slopes that are equal; perpendicular lines have slopes that are opposite reciprocals (the product of the slopes is -1).*
 - *The Segment Addition Postulate states that $AB + BC = AC$ if B is between A and C and the points are collinear. If the points are not collinear, then $AB + BC > AC$.*
- *If desired, you may explore with the class which of the statements in the activity are also biconditional statements (definitions that are always true).*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "8744" in the quick search box.***

Associated Materials

- *Conditional_Student.doc*
- *Conditional.tns*
- *Conditional_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.



- *Preparing for Proofs (TI-84 Plus and TI-Navigator) — 7769*
- *Congruent Triangles (TI-Nspire technology) — 8516*

Problem 1 – Slopes of lines

On page 1.3, students are to construct a line using the **Line** tool (**MENU > Points & Lines > Line**) and a point not on the line using the **Point** tool (**MENU > Points & Lines > Point**).

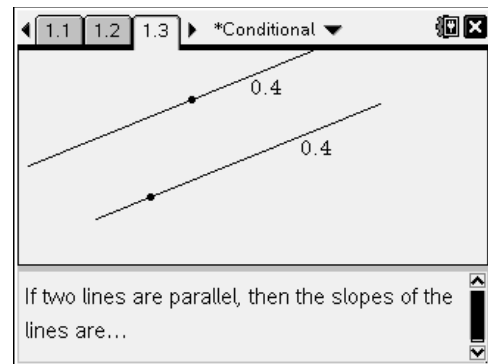
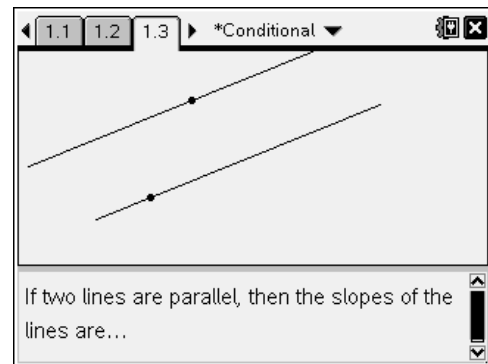
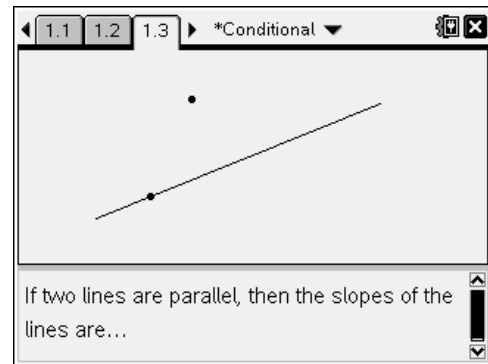
Next, students will use the **Parallel** tool (**MENU > Construction > Parallel**) to construct a new line through the point parallel to the original line.

Now students should find the slope of each line by selecting **MENU > Measurement > Slope**.

Note: You must press  (or **enter**) once to select the line whose slope you wish to show and then press  (or **enter**) again to anchor the measurement.

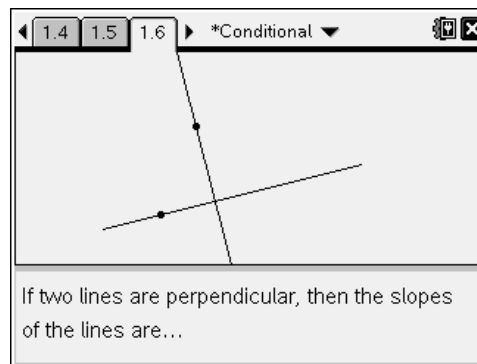
They can now drag the original line or the point and observe the results. Ask: What is true of the slopes of parallel lines?

Students should record their observations on page 1.4 and on the worksheet for part **A**. They will write the converse, inverse, and contrapositive of the statement and determine the truth of each.



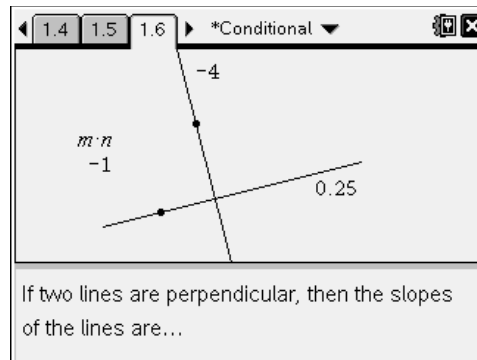
TI-Nspire Navigator Opportunity: Quick Poll
See Note 1 at the end of this lesson.

On page 1.6, students will construct a line and a point not on the line. They will then construct a perpendicular line to the original line through the point using the **Perpendicular** tool (**MENU > Construction > Perpendicular**).



Next, students should find the slope of both lines. Students can then drag the original line or the point and observe the results.

Have them select **MENU > Tools > Text** and enter $m \cdot n$ on the screen. They can use this with the **Calculate** tool (**MENU > Tools > Calculate**) to find the product of the slopes. Again, students should drag the line to observe the results. Ask: What is true of the slopes of perpendicular lines?

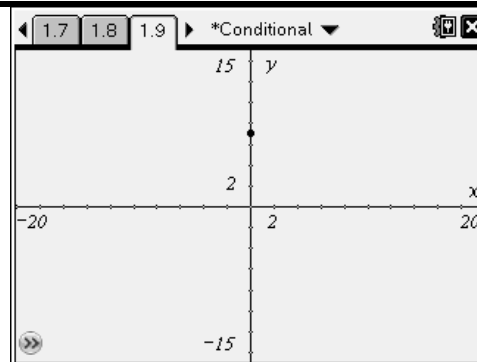


Students should record their observations, write conditional statements, and determine the truth of each on the worksheet part **B** and page 1.7.

TI-Nspire Navigator Opportunity: Screen Capture

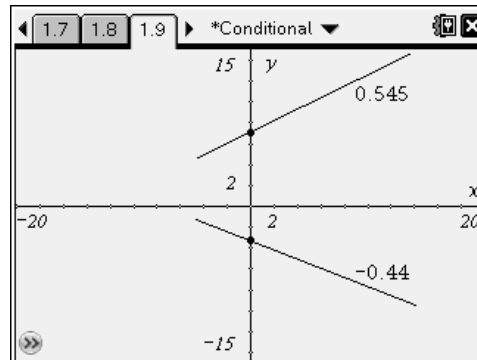
See Note 2 at the end of this lesson.

On page 1.9, students should first draw a point on the y -axis by selecting **MENU > Points & Lines > Point On**.



Next, they will construct two different lines with the same y -intercept (the point previously created on the y -axis) and display the slopes of both lines.

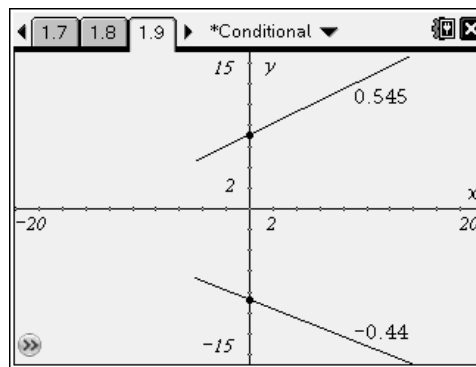
Note: If desired, students can display the equations of both lines using the **Coordinates and Equations** tool (**MENU > Tools > Coordinates and Equations**) instead of the slopes.



Students will then drag the lines and the y -intercept point and observe the changes in the coordinates and slopes.

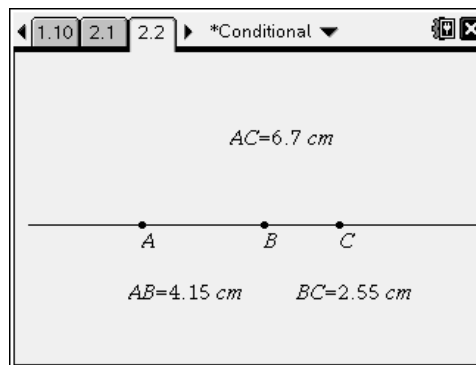
Students will write conditional statements on the worksheet part **C**. They should sketch counterexamples for any false statements.

The screenshot at right shows a counterexample for the converse statement: "If two lines have different slopes, then they have the same y -intercept."



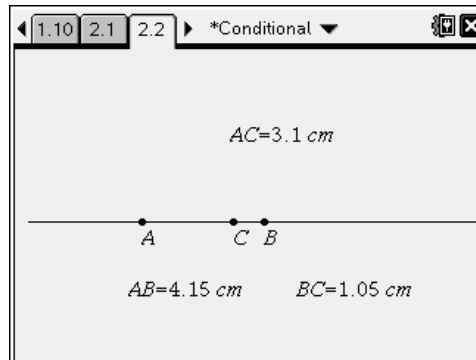
Problem 2 – Collinear and noncollinear segments

On page 2.2, points A , B , and C are constructed on a line and the lengths AB , BC , and AC are displayed.



Students will drag each of the points and observe the changes in the lengths. Be sure that students drag all three points and change the order of the points (they should investigate situations where B is not between the other two points).

Have them record their observations on the worksheet and write a conditional statement.



TI-Nspire Navigator Opportunity: Live Presenter

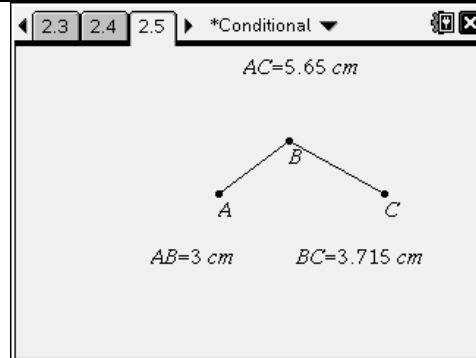
See Note 3 at the end of this lesson.

On page 2.5, points A , B , and C are not collinear. The lengths AB , BC , and AC are displayed.

Students should now drag the points and observe the changes in the distances.

If desired, students can use the **Text** tool and the **Calculate** tool to find the sum of $AB + BC$.

Students should again record observations on the worksheet and write a conditional statement.



Solutions – Student Worksheet**Problem 1**

- A. If two lines are parallel, then the slopes of the lines are equal.

Converse: If the slopes of the lines are equal, then the two lines are parallel.

Inverse: If the two lines are not parallel, then the slopes of the lines are not equal.

Contrapositive: If the slopes of the lines are not equal, then the two lines are not parallel.

Each conditional statement is true.

- B. If two lines are perpendicular, then the slopes of the lines are equal to -1 .

Converse: If the slopes of two lines are equal to -1 , then the lines are perpendicular.

Inverse: If two lines are not perpendicular, then the slopes of the lines are not equal to -1 .

Contrapositive: If the slopes of two lines are not equal to -1 , then the lines are not perpendicular.

Each conditional statement is true.

- C. *Converse:* If two lines have different slopes, then the lines have the same y -intercept.

Inverse: If two different lines do not have the same y -intercept, then the lines do not have different slopes.

Contrapositive: If two lines do not have different slopes, then the different lines do not have the same y -intercept.

Each conditional statement is false.

Problem 2

- A. Answers will vary for distances of AB , BC , and AC .

The lengths AB and BC add up to equal AC when B is between A and C .

If A , B , and C are collinear and B is between A and C , then $AB + BC = AC$.

- B. Answers will vary for distances of AB , BC , and AC .

$$AB + BC > AC$$

If A , B , and C are not collinear, then the sum of the lengths of AB and BC is greater than the length of AC .

TI-Nspire Navigator Opportunities**Note 1****Problem 1, *Quick Poll***

Have students answer the question at the bottom of page 1.4. You can see if the whole class found that the slopes are equal.

Note 2**Problem 1 page 1.6, *Screen Capture***

Use Screen Capture to ensure students are performing steps in the construction correctly.

Note 3**Problem 2, *Live Presenter***

Assign a student as the Live Presenter when they are dragging points A, B, and C. Give them instructions on where to move a specific point and have a class discussion about what students think will happen to the related values.