Limits Asymptotes Rational Functions

TI PROFESSIONAL DEVELOPMENT   TEACHER NOTES AND SOLUTIONS

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1. We are going to investigate what and where the horizontal and vertical asymptotes are for:

$$y = \frac{2x + 3}{x - 1.}$$

Notice the decimal point after the '1' in the denominator.

This will ensure that the values displayed in the table will be decimals and not fractions.

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2. Enter the rational function into y1 but do not graph it. We will analyze the equation first.

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3. We will use the *Table Ask* feature and the concept of limits to do so.

Press [2nd] [window] and set the Table as shown below:

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4. Type the following values for x in the table as shown below:

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5. Explain what these 3 points mean in the graph of

$$y = \frac{2x + 3}{x - 1.}$$

(0, − 3) is the y-intercept

(− 1.5, 0) is the x-intercept

At x = 1, the function is undefined.

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6. Let's investigate what is occurring 'around' x = 1.
First delete the 3 ordered pairs in the table by pressing on [del] for each x-value.
This clears the table.

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7. Let's first look at values of x that are getting closer and closer to 1, but larger than 1.

That is, $\lim\limits_{x \to 1^{+}} \left(\dfrac{2x + 3}{x - 1.} \right) = \ ?$

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8. Type in the following values for x in the newly empty table.

Press [enter] after each value and notice what is happening with the y-values.

$x = 1.1,\ 1.01,\ 1.001,$
$1.0001,\ 1.00001$

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9. Continue investigating by typing the following values and press [enter] after each value. Notice what is happening with the y-values.
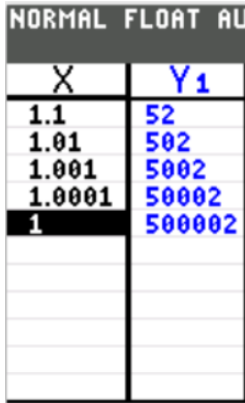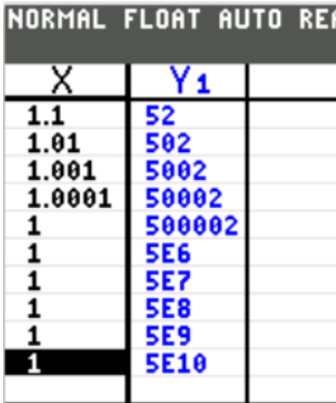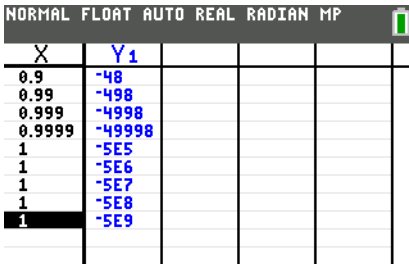
$x = 1.0000001,\ 1.00000001,$
$1.000000001,\ 1.0000000001$

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	8. continued	9. continued
	NORMAL FLOAT AU X | Y₁ 1.1 | 52 1.01 | 502 1.001 | 5002 1.0001 | 50002 **1** | 500002 X=1.00001	NORMAL FLOAT AUTO RE X | Y₁ 1.1 | 52 1.01 | 502 1.001 | 5002 1.0001 | 50002 1 | 500002 1 | 5E6 1 | 5E7 1 | 5E8 1 | 5E9 **1** | 5E10 X=1.0000000001
10. Based on the answers to steps 11 and 12, explain what is happening as $x \to 1^{+}$. **As x approaches 1 from the right, or from the positive side, y INcreases without bound,** $y \to +\infty$.	11. Delete these ordered pairs in the table by pressing on $\boxed{\text{del}}$ for each x-value. Let's look at values of x that are close to 1 , but smaller than 1. That is, $\lim\limits_{x\to 1^{-}}\left(\dfrac{2x+3}{x-1.}\right) = ?$	12. Type in the following values for x and press $\boxed{\text{enter}}$ after each value. Notice the y-values. $x = 0.9,\ 0.99,\ 0.999,\ 0.9999$ $0.99999,\ 0.999999,\ 0.9999999$ $0.99999999,\ 0.999999999$ NORMAL FLOAT AUTO REAL RADIAN MP X | Y₁ 0.9 | ⁻48 0.99 | ⁻498 0.999 | ⁻4998 0.9999 | ⁻49998 1 | ⁻5E5 1 | ⁻5E6 1 | ⁻5E7 1 | ⁻5E8 **1** | ⁻5E9 X=0.999999999
13. Based on the answers to step 15, explain what is happening as $x \to 1^{-}$. **As x approaches 1 from the left, or from the negative side, y DEcreases without bound,** $y \to -\infty$.	14. Next we will investigate what is happening as x increases without bound, that is, $\lim\limits_{x\to +\infty}\left(\dfrac{2x+3}{x-1.}\right) = ?$	15. Type in the following values for x and press $\boxed{\text{enter}}$ after each value. Notice the y-values. $x = 10,\ 100,\ 1000,\ 10^{4},$ $10^{5},\ 10^{6},\ 10^{7},\ 10^{8},\ 10^{9}$ Explain what is happening as $x \to +\infty$.
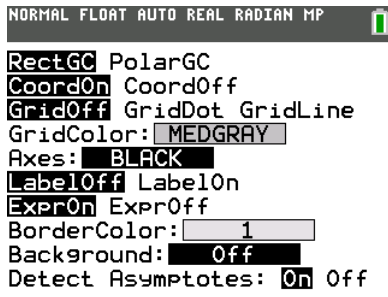
15. continued

X	Y1
10	2.5556
100	2.0505
1000	2.005
10000	2.0005
100000	2.0001
1E6	2
1E7	2
1E8	2
1E9	2

Y1=2.000000005

As x INcreases without bound, y gets close to 2, but a bit larger than 2.

$y \rightarrow 2^+$

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16. Investigate what is happening as x decreases without bound, that is,

$$\lim_{x \to -\infty} \left(\frac{2x + 3}{x - 1.} \right) = ?$$

Type in the following values for x and press enter after each value. Notice the y-values.
$x = -10, \ -100, \ -1000,$

$-10^4, \ -10^5, \ -10^6, \ -10^7$

$-10^8, \ -10^9$

17. Press [2nd] [zoom] to turn **On** the *Detect Asymptotes* feature.

NORMAL FLOAT AUTO REAL RADIAN MP

RectGC PolarGC
CoordOn CoordOff
GridOff GridDot GridLine
GridColor: MEDGRAY
Axes: BLACK
LabelOff LabelOn
ExprOn ExprOff
BorderColor: 1
Background: Off
Detect Asymptotes: On Off

Graph the function.

18. Compare the graph to what you discovered in your analysis.

a) State the equation of the vertical asymptote and explain what is occurring around it.

x = 1 is the equation of the vertical asymptote.

As x gets close to 1 from the left side, the function is going down infinitely.

As x get close to 1 from the right side, the function is going up infinitely.

16. continued

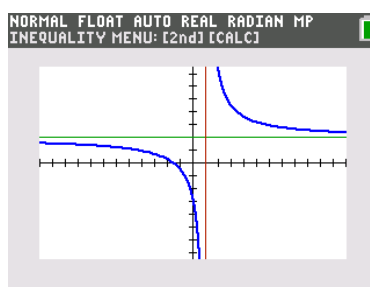| X	Y₁
-10	1.5455
-100	1.9505
-1000	1.995
-10000	1.9995
-1E5	2
-1E6	2
-1E7	2
-1E8	2
-1E9	2

$Y_1 = 1.999999995$

Explain what is happening as $x \to +\infty$.

As x DEcreases without bound, y gets close to 2, but a bit smaller than 2.

$y \to 2^-$

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17. continued

NORMAL FLOAT AUTO REAL RADIAN MP
INEQUALITY MENU: [2nd] [CALC]

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18. continued

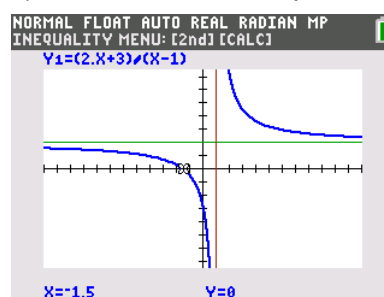b) State the equation of the horizontal asymptote and explain what is occurring around it.

y = 2 is the equation of the horizontal asymptote.

As x gets very, very large, the y value gets very close to 2, but just above 2.

As x gets very, very small, the y value gets very close to 2, but just below 2.

c) Trace to the x-intercept.

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INEQUALITY MENU: [2nd] [CALC]
Y1=(2.X+3)/(X-1)
X=-1.5   Y=0

d) Trace to y-intercept.

NORMAL FLOAT AUTO REAL RADIAN MP
INEQUALITY MENU: [2nd] [CALC]
Y1=(2.X+3)/(X-1)
X=0   Y=-3