Implicit Differentiation

ImplicitDifferentiation.tns

Name ______________________

Class ______________________

On page 1.2, the diagram shows a circle with equation $x^2 + y^2 = 36$. Suppose you were asked to find the slope of the curve at $x = 2$.

- Why might this question be potentially difficult to answer?
- What strategies or methods could you use to answer this question?

Problem 1 – Finding the derivative of $x^2 + y^2 = 36$

The relation $x^2 + y^2 = 36$ in its current form *implicitly* defines two functions, $f_1(x) = y$ and $f_2(x) = y$. Move to page 1.3. Find these two functions by solving $x^2 + y^2 = 36$ for y.

$f_1(x) =$ $\qquad\qquad$ $f_2(x) =$

This confirms that $f_1(x)$ and $f_2(x)$ *explicitly* define the relation $x^2 + y^2 = 36$.

One way to find the slope of a tangent drawn to the circle at any point (x, y) located on the circle is by finding the derivative of $f_1(x)$ and $f_2(x)$.

$\frac{dy}{dx} f_1(x) =$ $\qquad\qquad$ $\frac{dy}{dx} f_2(x) =$

On page 1.4, enter the equation of your derivative for $f_1(x)$ to see if its graph matches the graph of the derivative displayed on the right side of the screen.

On page 1.5, determine the slopes of the tangents to $x^2 + y^2 = 36$ at $x = 2$.

Note: The Math Box is set up to evaluate the derivative of $f_1(x)$ at $x = 2$ (***f***(*x*) from page 1.4). After determining the slope of the tangent to $f_1(x)$, edit the command to determine the slope of the tangent to $f_2(x)$ at $x = 2$.

$\frac{dy}{dx} f_1(2) =$ $\qquad\qquad$ $\frac{dy}{dx} f_2(2) =$

Another way to find the slopes of the tangents is by finding the derivative of $x^2 + y^2 = 36$ using *implicit differentiation*. Advance to page 1.6 and evaluate the Math Boxes by pressing enter in the Math Box. On a *Calculator* application page, you can find the **impDif** command by selecting **MENU > Calculus > Implicit Differentiation**. Enter **impDif($x^2 + y^2$ = 36, *x*, *y*)** to find dy/dx.

$$\frac{dy}{dx} =$$

Use this result to find the slopes of the tangents to $x^2 + y^2 = 36$ at $x = 2$.

- Are your answers consistent with those found earlier?

- Rewrite the implicit differentiation derivative in terms of *x*. Show that, for all values of *x* and *y*, the derivatives of $f_1(x)$ and $f_2(x)$ that you found earlier are the same as those found using the **impDif** command.

Problem 2 – Performing implicit differentiation by hand

Move to page 2.1. To find the derivative of a relation $F(x, y)$, take the derivative of *y* with respect to *x* of each side of the relation. Looking at the original example, $x^2 + y^2 = 36$, we get:

$$\frac{d}{dx}\left(x^2+y^2\right)=\frac{d}{dx}(36) \quad \rightarrow \quad \frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}(36)$$

Evaluate the following by hand.

$$\frac{d}{dx}\left(x^2\right)= \qquad\qquad \frac{d}{dx}(36)=$$

Use the **Derivative** command to find $\frac{d}{dx}\left(y^2\right)$. Set up the expression up as $\frac{d}{dx}\left(\left(y(x)\right)^2\right)$. Notice that $y(x)$ is used rather than just *y*. This is very important because it reminds the device that *y* is a function of *x*.

$$\frac{d}{dx}\left(y^2\right)=$$

You have now evaluated $\frac{d}{dx}\left(x^2\right)$, $\frac{d}{dx}\left(y^2\right)$, and $\frac{d}{dx}(36)$. Replace these expressions in the equation $\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}(36)$ and solve for $\frac{dy}{dx}$.

Compare your result to the one obtained using the **impDif** command.

Problem 3 – Finding the derivative of $y^2 + xy = 2$

Move to page 3.1. The relation $y^2 + xy = 2$ can also be solved as two functions, $f_1(x)$ and $f_2(x)$, that *explicitly* define it.

- What strategy can be used to solve $y^2 + xy = 2$ for y by hand?

- Solve $y^2 + xy = 2$ for y and use the **Solve** command to check your answer. (On a Notes page, this command is found under **MENU > Calculations > Algebra > Solve.**)

The derivative of $y^2 + xy = 2$ can then be found by finding the derivatives of $f_1(x)$ and $f_2(x)$. However, the derivative can be found more easily using implicit differentiation.

Use implicit differentiation to find the derivative of $y^2 + xy = 2$. Check your result by using the **impDif** command on page 3.2. (Hint: The product rule must be used to find the derivative of xy.)

$\frac{dy}{dx} =$

You can also verify your result graphically. Advance to page 3.4, which displays a graph of $y^2 + xy = 2$. Tangents are drawn to the curve and their corresponding slopes are shown on the screen.

- Use the derivative you found for $y^2 + xy = 2$ to calculate the slopes at $x = -6$.

- Do your calculations give the same slopes shown as m_1 and m_2 on the screen?

- Drag the open circle on the x-axis and check your result for a different ordered pair.

Extension – Finding the derivative of $x^3 + y^3 = 6xy$

The relation $x^3 + y^3 = 6xy$ cannot be solved explicitly for y. In this case implicit differentiation must be used.

- Find the derivative of $x^3 + y^3 = 6xy$ and use the **impDif** command on the Notes page on 4.1 and use the 'show' slider on page 4.2 to graphically verify your result.

  $$\frac{dy}{dx} =$$

- Use this result to find the slopes of the tangents to $x^3 + y^3 = 6xy$ at $x = 1$.
  (<u>Hint</u>: Use the **Solve** command to find the y-values that correspond to $x = 1$.)

