

Activity 7

Now You See It, Now You Don't

Objectives

- ◆ To study the relationship between age and near point accommodation
- ◆ To predict a person's age based on near point accommodation
- ◆ To use technology to study an exponential regression
- ◆ To use technology to create a box-and-whisker plot
- ◆ To use technology to create a histogram

Materials

- ◆ TI-73 graphing device
- ◆ Metric ruler or meter stick
- ◆ String length: 1.5 meters

Introduction

Have you ever noticed someone holding an object away from his or her face so that they can see it more easily? This may seem somewhat perplexing. Why would holding an object *further* away make it *easier* to examine?

Focusing one's eye when looking at a close object is referred to as *near point accommodation*. The focusing of one's eye is made possible by little muscles that pull on the eye, and therefore slightly reshape the lens. This causes the lens to focus the light on the sensitive cells of the retina.

Problem

Is near point accommodation related to a person's age?

Collecting the data

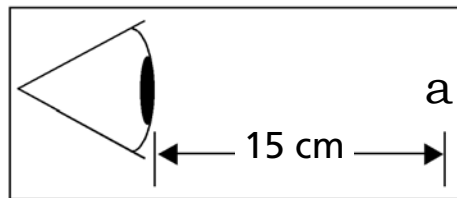
Follow steps 1 through 4 to determine a person's near point accommodation. Collect data at home using subjects in the 35 - 64 age group.

1. Have your subject hold the letter "a", shown at the right, in front of his/her face. Move the page as close to the subject as possible, keeping the letter in focus.

a

Note: If your subject normally wears glasses, then the glasses should be worn during this test (unless the glasses are reading glasses, in which case the glasses should not be worn). If your subject wears bifocals, then the subject should wear his / her glasses and should view the letter through the upper part of the glasses that are used for distance. If your subject wears separate glasses for distance and reading, then he / she should wear the glasses that are used for distance.

- Use a string to determine the distance from the front of the eye to the letter "a". **Do not hold the string too close to the eye.** Holding the string along your subject's face, extend the string from the cornea (the front part of the eye) to the page with the letter "a". In the example shown here, the near point accommodation is 15 centimeters.



- Record the result on the **Data Collection and Analysis** page. Be sure to indicate the age of the subject. Repeat this procedure with four other subjects.
- Submit your data to your teacher. Your teacher will collate the data from all members of the class.

Setting up the TI-73

Before starting your data collection, make sure that the TI-73 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-73

- Press **[LIST]**.
- Enter the age of the subjects in **L1**.
- Enter the near point accommodation of all the subjects in **L2**. (Make sure that the pairs of age and near point accommodation data match in each column.)
- Enter the near point accommodation of only the 35 - 44 age group subjects in **L3**.
- Enter the near point accommodation of only the 45 - 54 age group subjects in **L4**.
- Enter the near point accommodation of only the 55 - 64 age group subjects in **L5**.

| L1 | L2 | L3 | 1 |
|----------|-------|-------|---|
| ████████ | ----- | ----- | |
| L1(1)= | | | |

| L1 | L2 | L3 | 3 |
|---------|----|----------|---|
| 35 | 14 | ████████ | |
| 35 | 16 | | |
| 36 | 15 | | |
| 38 | 16 | | |
| 40 | 20 | | |
| 42 | 18 | | |
| 44 | 28 | | |
| L3(1) = | | | |

| L3 | L4 | L5 | 5 |
|------------|----|----------|---|
| 14 | 25 | ████████ | |
| 16 | 29 | 61 | |
| 15 | 34 | 80 | |
| 16 | 32 | 72 | |
| 20 | 38 | 91 | |
| 18 | 53 | 108 | |
| 28 | 57 | 120 | |
| L5(1) = 58 | | | |

Setting up the window

1. Press **WINDOW** to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ΔX Value.** Set the **Xscl** to **2**.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **10**.

```

WINDOW
Xmin=30
Xmax=65
 $\Delta$ X= 3723404255...
Xscl=2
Ymin=0
Ymax=180
Yscl=10
  
```

Graphing the data: Setting up a scatter plot

You can analyze the data in several different ways. You will need to set up a scatter plot and model the data (exponential regression). You can then use the data collected to predict a person's age based on their near point accommodation.

1. Press **2nd** **[PLOT]**. Select **1:Plot1** by pressing **1** or **ENTER**.

```

STAT PLOTS
1:Plot1...Off
  L1 L2
2:Plot2...Off
  L3 L4
3:Plot3...Off
  L1 L2
4↓PlotsOff
  
```

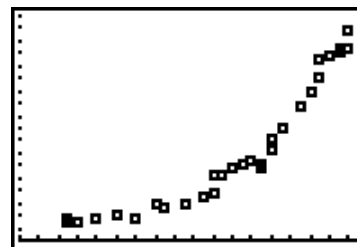
2. Set up the plot as shown by pressing **ENTER**
ENTER **2nd** **[STAT]** **1:L1** **2nd** **[STAT]** **2:L2**
ENTER.

```

Plot1 Off
Type:
Xlist:L1
Ylist:L2
Mark:
  
```

3. Press **GRAPH** to see the plot.

Observe the shape of the plot. It is not linear because the slope changes. What type of regression would model such data?



Answer questions 1 and 2 on the **Data Collection and Analysis** page.

Analyzing the data

Finding a best fit line

1. Find an exponential model for the data.
Press 2nd $[\text{STAT}]$ \downarrow to move the cursor to the **Calc** menu.

```

Ls OPS MATH [2] [0]
1:1-Var Stats
2:2-Var Stats
3:Manual-Fit
4:Med-Med
5:LinReg(ax+b)
6:QuadReg
7:ExpReg
  
```

2. Select **7:ExpReg** by pressing **7**.

```

ExpReg
  
```

3. Press 2nd $[\text{STAT}]$ **1:L1** $,$ 2nd $[\text{STAT}]$ **2:L2** $,$.

```

ExpReg L1,L2, [ ]
  
```

4. Press 2nd $[\text{VARS}]$. Select **2:Y-Vars** by pressing **2**.

```

FUNCTIO
1:Y1
2:Y2
3:Y3
4:Y4
5:FnOn
6:FnOff
  
```

5. Select **1:Y1** by pressing **1** or $[\text{ENTER}]$.

```

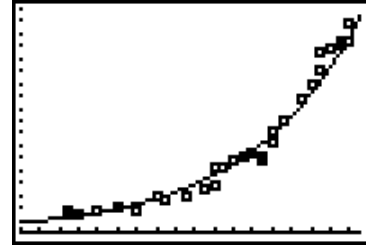
ExpReg L1,L2,Y1 [ ]
  
```

6. Press $[\text{ENTER}]$ to calculate the exponential regression. The function is pasted in **Y1**.

```

ExpReg
y=a*b^x
a=.5539307996
b=1.092480652
[ ]
  
```

7. Press **GRAPH** to see the exponential regression model.



Answer questions 3 and 4 on the **Data Collection and Analysis** page.

Determining age based on the near point accommodation

If a person has a near point accommodation of 47 cm, how old is that person likely to be according to the data?

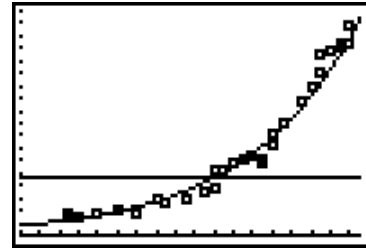
1. Press **Y=** and move the cursor to **Y2**. Enter **47**, the near point accommodation.

```

Plot1 Plot2 Plot3
\Y1= .55393079963
669*1.0924806524
581^X
\Y2= 47
\Y3=
\Y4=

```

2. Press **GRAPH** to see the intersection of the two functions. The x value of the point where the two functions intersect is the predicted age of the person if their near point accommodation is 47 cm.



The table function of the TI-73 will be used to determine the coordinates of the point of intersection.

3. Press **2nd** [TBLSET]. Type the lowest value in **L1** (35 in the example). Press **5** to set the **ΔTbl** value.

```

TABLE SETUP
TblStart=35
ΔTbl=5
Indent: Auto Ask
Depend: Auto Ask

```

4. Press **2nd** [TABLE]. If necessary, use **↓** and **↑** to scroll the table.

Note: For this example, in the **Y1** column, 47 cm falls between 46.147 and 71.814 which corresponds to ages 50 and 55. Based on this information, the table will be readjusted.

| X | Y1 | Y2 |
|----|--------|----|
| 35 | 12.244 | 47 |
| 40 | 19.055 | 47 |
| 45 | 29.653 | 47 |
| 50 | 46.147 | 47 |
| 55 | 71.814 | 47 |
| 60 | 111.76 | 47 |
| 65 | 173.92 | 47 |

X=35

5. Press $\boxed{2\text{nd}}$ [TBLSET]. Enter the result from Step 4 in **TblStart**. Press $\boxed{\downarrow}$ **0.1** to set the ΔTbl value.



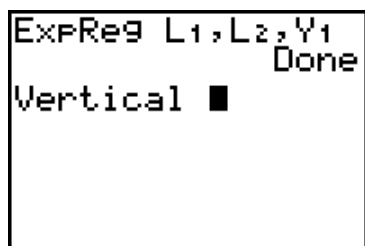
6. Press $\boxed{2\text{nd}}$ [TABLE]. If necessary, use $\boxed{\downarrow}$ and $\boxed{\uparrow}$ to scroll the table.

Note: The data used to construct the exponential model used ages rounded to the nearest year. We should report age to the same level of precision. From the table, 47 falls between 46.971 and 47.388 which corresponds to 50.2 and 50.3. Rounding to the nearest year, the intersection point is (50, 47).

| X | Y1 | Y2 |
|------|--------|----|
| 50 | 46.147 | 47 |
| 50.1 | 46.557 | 47 |
| 50.2 | 46.971 | 47 |
| 50.3 | 47.388 | 47 |
| 50.4 | 47.809 | 47 |
| 50.5 | 48.234 | 47 |
| 50.6 | 48.662 | 47 |

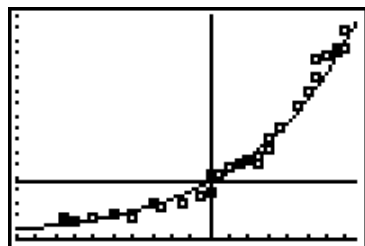
X=50

7. To verify this graphically, use the **DRAW** function. Press $\boxed{\text{DRAW}}$. Select **4:Vertical** by pressing **4**.



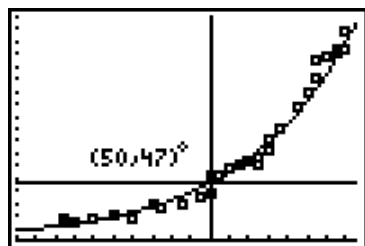
8. Type your results from Step 6 (for this example, **50**) and press $\boxed{\text{ENTER}}$.

Note: The coordinates of the point on the exponential model where all of the curves intersect is defined for this example by the vertical drawn at $x=50$ and the horizontal at $y=47$.



9. The coordinates of the intersection can be added onto the screen by pressing $\boxed{\text{DRAW}}$ **7:Text**, moving the cursor near the point of intersect and typing your results.

Note: Text appears below and to the right of the cursor.



Answer question 5 on the **Data Collection and Analysis** page.

Graphing the data: Setting up a box-and-whisker plot

You can use a box-and-whisker plot to analyze the near point accommodation of the different age groups. You have already entered the near point accommodations of 35 - 44 year olds in **L3**, 45 - 54 year olds in **L4**, and 55 - 64 year olds in **L5**.

1. Press **2nd** [PLOT]. Select **1:Plot1** by pressing **1** or **ENTER**.

```
STAT PLOTS
1:Plot1...On
  L1  L2  □
2:Plot2...Off
  L3  L4  +
3:Plot3...Off
  L1  L2  □
4↓PlotsOff
```

2. Set up the plot as shown by pressing **ENTER** **↓** **↓** **↓** **↓** **↓** **↓** **↓** **ENTER** **↓** **2nd** [STAT] **3:L3**. Press **↓** **1** to set the frequency.

```
Plot1  Off
Type:     
Xlist:L3
Freq:1
```

3. Press **2nd** [PLOT]. Select **2:Plot2** by pressing **2**.

```
STAT PLOTS
1:Plot1...On
  L3  1
2:Plot2...Off
  L3  L4  +
3:Plot3...Off
  L1  L2  □
4↓PlotsOff
```

4. Set up the plot as shown by pressing **ENTER** **↓** **↓** **↓** **↓** **↓** **↓** **↓** **ENTER** **↓** **2nd** [STAT] **4:L4**. Press **↓** **1** to set the frequency.

```
Plot2  Off
Type:     
Xlist:L4
Freq:1
```

5. Press **2nd** [PLOT]. Select **3:Plot3** by pressing **3**.

```
STAT PLOTS
1:Plot1...On
  L3  1
2:Plot2...On
  L4  1
3:Plot3...Off
  L1  L2  □
4↓PlotsOff
```


Data Collection and Analysis

Name _____

Date _____

Activity 7: Now You See It, Now You Don't

Near Point Accommodation Data: Home Survey

Collecting the data

| Age | Near Point Accommodation |
|-----|--------------------------|
| | |
| | |
| | |
| | |
| | |
| | |
| | |

| Age Group | Minimum | 1 st Quartile | Median | 3 rd Quartile | Maximum |
|--------------|---------|--------------------------|--------|--------------------------|---------|
| 35 - 44 (L3) | | | | | |
| 45 - 54 (L4) | | | | | |
| 55 - 64 (L5) | | | | | |

Analyzing the data

1. Describe the shape of the near point accommodation versus age plot. Is there a correlation between age and near point accommodation? Explain.

2. In which age group is the near point accommodation rising fastest: 35 - 44, 45 - 54, or 55 - 64? How can you tell by looking at the plot?

3. Write the exponential regression equation.

4. Does the exponential model seem to fit your data? Explain. (Does it seem to fit some age groups better than others?)

5. A person with a near point accommodation of 47 cm is likely to be _____ years old according to the exponential model.

6. What is the *median* near point accommodation of the three age groups analyzed?

Ages 35-44: _____ Ages 45-54: _____ Ages 55-64: _____

7. Observe the width of the three box-and-whisker plots. Which plot has the largest width? Which plot has the smallest width? How do these widths relate to the exponential regression analyzed earlier? (**Hint:** Each box-and-whisker plot represents the same number of years – 10.)

8. Do the box-and-whisker plots overlap? What does this tell you about the near point accommodations of the three age groups that were analyzed?

Extension

Use a histogram to analyze the extent to which near point accommodation varies within your age group. The following table shows some sample data (nearest 0.1 cm).

| | | | | | | | | | | | | | | | |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 7.1 | 6.5 | 9.3 | 6.2 | 8.7 | 9.0 | 7.7 | 8.1 | 8.3 | 7.8 | 8.9 | 8.8 | 6.9 | 9.1 | 7.5 | 8.3 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|

Examine the distribution of near point accommodations in your class.

Which near point accommodations are least common in your age group? Most common?

How could you adjust the window to study different distribution ranges within your age group?

Teacher Notes



Activity 7

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Objectives

- ◆ To study the relationship between age and near point accommodation
- ◆ To predict a person's age based on near point accommodation
- ◆ To use technology to study an exponential regression
- ◆ To use technology to create a box-and-whisker plot
- ◆ To use technology to create a histogram

Materials

- ◆ TI-73 graphing device
- ◆ Metric ruler or meter stick
- ◆ String length: 1.5 meters

Preparation

- ◆ You may choose to use the near point accommodation data provided. If you decide to have your class collect the data, there are several ways to collate it. One option is to type the near point accommodation data directly into the TI-73 in **L1** and **L2** and then share the data with the students by linking calculators. You may want to print the data on a handout or put it on the chalkboard and have the students copy it into their lists.
- ◆ Real studies relating age and near point accommodation (presbyopia) show a general increase with age, but the relationship is not neatly modeled mathematically. From ages 35 - 64, it is close to an exponential model.

Answers to Data Collection and Analysis questions

Collecting the data

Sample data:

| Age (years) – L1 | Near point (cm) – L2 | Age (years) – L1 | Near point (cm) – L2 |
|---------------------|-------------------------|---------------------|-------------------------|
| 35 | 14 | 54 | 65 |
| 35 | 16 | 55 | 58 |
| 36 | 15 | 55 | 61 |
| 38 | 16 | 56 | 80 |
| 40 | 20 | 56 | 72 |
| 42 | 18 | 57 | 91 |
| 44 | 28 | 59 | 108 |
| 45 | 25 | 60 | 120 |
| 47 | 29 | 61 | 146 |
| 49 | 34 | 61 | 132 |
| 50 | 52 | 62 | 147 |
| 50 | 38 | 63 | 150 |
| 51 | 53 | 63 | 155 |
| 52 | 57 | 64 | 167 |
| 53 | 62 | 64 | 153 |

Analyzing the data

- Describe the shape of the near point accommodation versus age plot. Is there a correlation between age and near point accommodation? Explain.

The data will vary, but generally, one finds that it rises slowly at first and then more steeply in the upper age groups. As one's age rises, the near point accommodation rises, somewhat exponentially.

- In which age group is the near point accommodation rising fastest: 35 - 44, 45 - 54, or 55 - 64? How can you tell by looking at the plot?

One would expect the steepest rise to be in the 55 - 64 age group. This is where the plot is the steepest.

- Write the exponential regression equation.

Answers will vary depending on what data is used. For the given data, the regression is:

$$Y = (0.56) (1.1)^X$$

4. Does the exponential regression seem to fit your data? Explain. (Does it seem to fit some age groups better than others?)

Answers will vary depending on what data is used. Based on the data provided, the regression appears to fit the data, but clearly there are individuals that are off the regression line.

5. A person with a near point accommodation of 47 cm is likely to be ____ years old according to the exponential model.

Answers will vary depending on the data used. According to the data provided, the age of the individual is approximately 50. However, if one examines the actual data, it is obvious that there is a fair amount of variation in near point accommodations at any given age.

6. What is the *median* near point accommodation of the three age groups analyzed?

Answers will vary. For the sample data:

Ages 35 - 44: Median is 16 cm.

Ages 45 - 54: Median is 52 cm.

Ages 55 - 64: Median is 126 cm.

7. Observe the width of the three *box-and-whisker* plots. Which plot has the largest width? Which plot has the smallest width? How do these widths relate to the exponential model analyzed earlier? (**Hint:** Each box-and-whisker plot represents the same number of years – 10.)

The box-and-whisker plot is for a 10-year range. The higher the age, the wider the plot because in an exponential model for near point accommodation versus age, as the age increases, the near point accommodation values increase at a faster rate. For that age range, there is a greater amount of variation in near point accommodation values.

8. Do the box-and-whisker plots overlap? What does this tell you about the near point accommodations of the three age groups that were analyzed?

The box-and-whisker plots will probably overlap, showing that there are subjects in one age group that have higher near point accommodations than subjects in the next older age group.

