

## Activity 4

### The Calcumites are Coming!

#### Objectives

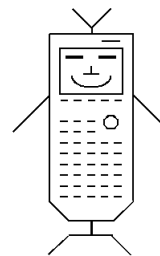
- ◆ To model the growth of a population
- ◆ To compare ideal population growth with a population whose growth is limited
- ◆ To use technology to find an exponential and logistic regression equation
- ◆ To use technology to plot an exponential and logistic model

#### Materials

- ◆ TI-83 Plus
- ◆ Calcumite cut outs, pennies, or candy

#### Introduction

This is a Calcumite. It may look cute to the casual observer, but it is a creature that threatens to take over the world. Mutant, evolutionary cousins of your graphing calculator, these calcumites are increasing in number and will perhaps one day control the whole world! Their motto is “you push my buttons and I’ll push yours.” Does that sound like a friendly critter to you? Our only hope lies in understanding the population dynamics of this creature; a formidable task that will be facilitated by our TI-83 Plus, the only weapon that may save us!



#### Problem

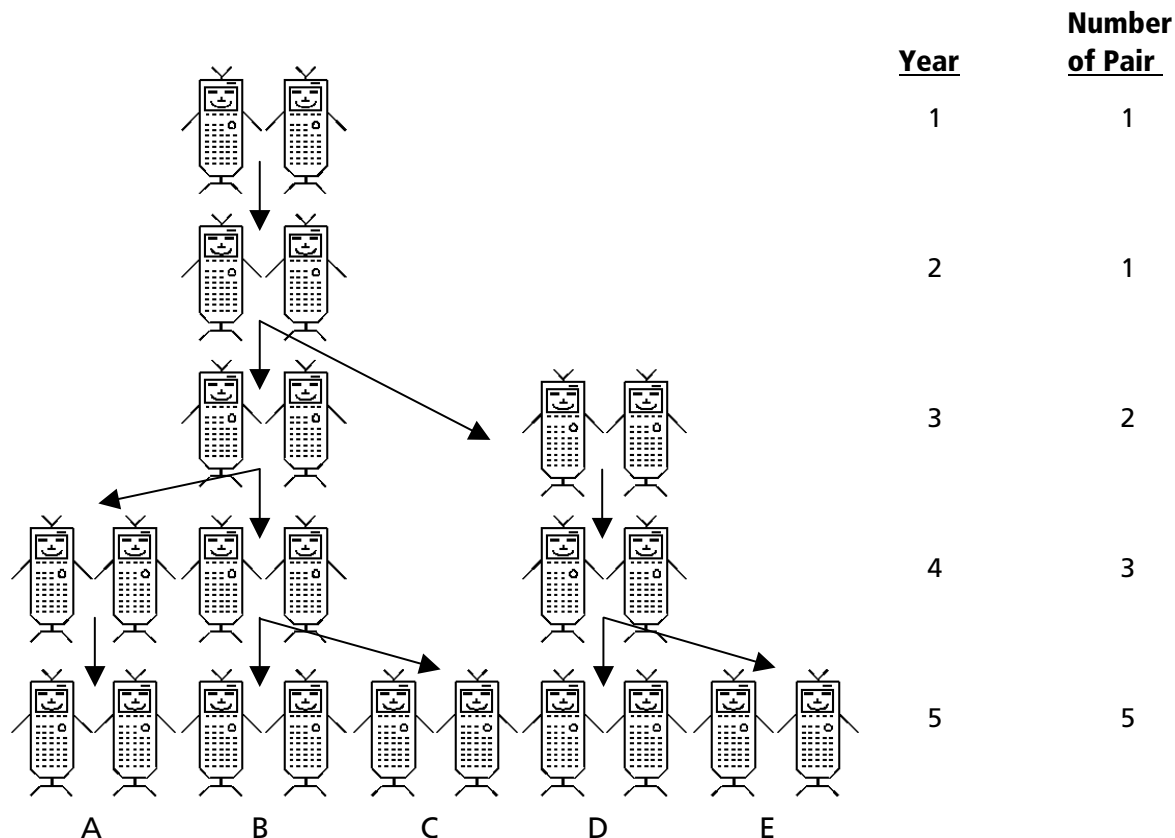
Understanding population dynamics as it relates to both humans and other organisms is important in understanding the balance of nature. Ideal growth of a population refers to a population whose growth is unimpeded in any way — not by predators, disease, overcrowding, or limited resources. In practice, ideal growth cannot continue indefinitely because there are limiting factors in nature. How can we mathematically model a population’s growth when it is *ideal* and when it is *limited*?

#### Collecting the data — Part I

In the first part of this activity, you will determine the growth rate of an *ideal* population.

1. The following page shows six generations of calcumites. Make the following assumptions about calcumites:
  - ◆ They mate when they are one year old.

- ◆ They mate once each year.
  - ◆ They produce one pair (a male and a female) each time they mate.
  - ◆ They never die.
2. The population for each year follows the sequence 1, 1, 2, 3, 5. To determine the next number in the sequence, carefully examine the diagram. Observe that all five pairs A, B, C, D, and E will all survive. In addition, 3 pairs, A, B, and D will have a pair of offspring, bringing the total pairs of calcumites for the next generation to 8.



Determine the relationship of numbers in this sequence. (It may be easier to work backwards.) How is the 8 related to the two integers before it? How is the 5 related to the two integers before it?

Predict the numbers after 8. Fill in the chart entitled *Ideal Population Growth — Exponential* on the **Data Collection and Analysis** page for the first 10 generations of calcumites. Check your answer with your teacher before proceeding to the next section.

### Setting up the TI-83 Plus

Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

### Entering the data in the TI-83 Plus

1. Press **[STAT]** and select **1:Edit** by pressing **[ENTER]**.
2. Enter the generation numbers 1 through 10 in **L1**.
3. Move the cursor to the right and enter the number of calcumite pairs in **L2**. (Make sure that the pairs of generation numbers and calcumite pairs match in each column.)

```

CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

L1	L2	3
1	1	-----
2	1	
3	2	
4	2	
5	3	
6	3	
7	13	
L3 =		

### Setting up the window

1. Press **[WINDOW]** to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to 1.
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to 5.

```

WINDOW
Xmin=0
Xmax=11
Xscl=1
Ymin=0
Ymax=60
Yscl=5
Xres=1
  
```

### Graphing the data: Setting up a scatter plot

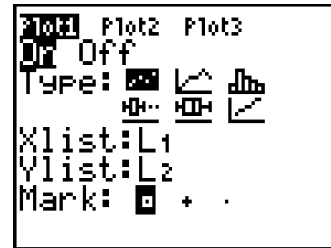
In order to analyze the data, you will need to set up a scatter plot and model the data by graphing a line of best fit (exponential regression).

1. Press **[2nd]** **[STAT PLOT]** and select **1:Plot1** by pressing **[ENTER]**.

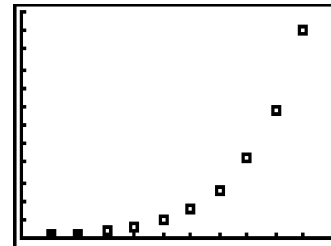
```

STAT PLOTS
1:Plot1...Off
  [ ] L1 L2 [ ]
2:Plot2...Off
  [ ] L1 RESID [ ]
3:Plot3...Off
  [ ] L1 L4 [ ]
4↓PlotsOff
  
```

- Set up the plot as shown by pressing  
 $\boxed{\text{ENTER}}$   $\boxed{\downarrow}$   $\boxed{\text{ENTER}}$   $\boxed{\downarrow}$   $\boxed{2\text{nd}}$   $\boxed{[L1]}$   $\boxed{\text{ENTER}}$   $\boxed{2\text{nd}}$   $\boxed{[L2]}$   
 $\boxed{\text{ENTER}}$   $\boxed{\text{ENTER}}$ .



- Press  $\boxed{\text{GRAPH}}$  to see the plot.



- The graph to the right shows the rate of calcumite population growth for three intervals of the plot shown in number 3:

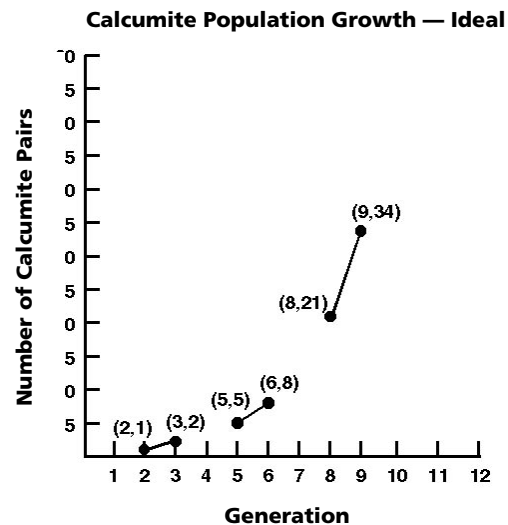
2 to 3 generations

5 to 6 generations

8 to 9 generations

Determine the growth rate for each of those time intervals by determining the *slope* of each of the plots, using the following formula for slope:

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



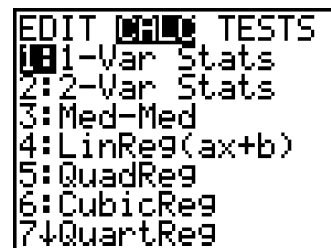
Enter the slopes on the **Data Collection and Analysis** page.

Answer Part I questions 1 and 2 on the **Data Collection and Analysis** page.

You have examined linear models in previous activities. In a linear model, the slope does not change. Therefore, a linear model would not be appropriate here. What type of regression would be a suitable model for this data?

**Analyzing the data: Finding an exponential regression**

- Press  $\boxed{\text{STAT}}$  and move the cursor to the **CALC** menu.



2. Press the up arrow  $\uparrow$  or the down arrow  $\downarrow$  to select **0:ExpReg** and press  $\text{ENTER}$ .

```

EDIT 0:00 TESTS
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg

```

```
ExpReg
```

3. Press  $\text{2nd}$   $[L1]$   $,$   $\text{2nd}$   $[L2]$   $,$ .

```
ExpReg L1,L2,
```

4. Press  $\text{VAR}$  and move the cursor to the **Y-VARS** menu.

```

VARS Y-VARS
1:Function...
2:Parametric...
3:Polar...
4:On/Off...

```

5. Select **1:Function** by pressing  $\text{ENTER}$ .

```

FUNCTION
1:Y1
2:Y2
3:Y3
4:Y4
5:Y5
6:Y6
7:Y7

```

6. Select **1:Y1** by pressing  $\text{ENTER}$ .

```
ExpReg L1,L2,Y1
```

7. Press  $\text{ENTER}$  to calculate the exponential regression and paste the function in **Y1**.

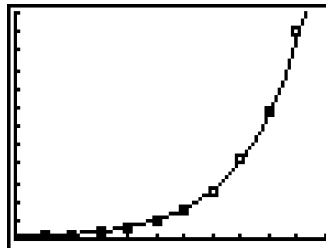
```

ExpReg
y=a*b^x
a=.4882960783
b=1.598311821

```

8. Press  $\text{GRAPH}$  to see the exponential regression.

The plot displayed in number 8 shows the calcumite population growth over a period of 10 generations. In order to examine how large the population will become after 20 generations, it is necessary to expand the window.



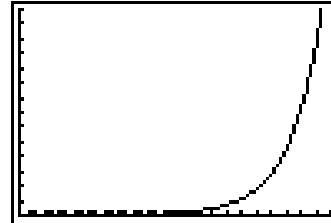
9. Press **WINDOW** and set the window values as shown.

```

WINDOW
Xmin=0
Xmax=21
Xscl=1
Ymin=0
Ymax=7000
Yscl=500
Xres=1

```

10. Press **GRAPH** to see the plot.



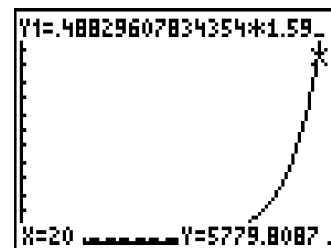
11. Press **2nd** **CALC** and select **1:value** by pressing **ENTER**.

```

MATH>
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

```

12. Type **20** and press **ENTER** to see the enlarged generation growth graph.



13. Record the number of pairs of calcumites after 20 generations on the **Data Collection and Analysis** page.

It becomes apparent that a population cannot keep rising at this rate. Keep in mind that a one-year generation time for calcumites is somewhat long compared to many other organisms. For example, the generation time of a fruit fly is about two weeks. Also, keep in mind that some organisms produce many offspring, not just one pair. For example, tapeworms may produce 600 million eggs annually, but only a few survive. There are always factors that limit the growth of a population. If this were not true, the earth would soon be covered with fruit flies, tapeworms, or our imaginary calcumites!

### Collecting the data — Part II

In the second part of this activity, you will determine the growth rate of a *limited* population.

As the calcumite population grows and becomes denser, more of them die for various reasons. Some die from disease. They are infected with a computer-like virus that attacks calcumites. TI-83 Plus calculators eat some of the calcumites.

Others perish because they cannot find enough food and resources. The following chart shows 10 generations of calcumites whose growth is *limited*. Plot the growth of this population and analyze the consequences of such limited growth. .

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	8	10	11	12	12

### Setting up the TI-83 Plus

Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

### Entering the data in the TI-83 Plus

1. Press **[STAT]** and select **1:Edit** by pressing **[ENTER]**.

```

[STAT] CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor
  
```

The generation data in **L1** is the same as in the study of ideal population growth and need not be changed.

2. Clear the data from **L2** and enter the number of calcumite pairs. Move the cursor to the **L2** label and press **[CLEAR]** **[ENTER]**.

L1	L2	L3	Z
5	5		
6	8		
7	10		
8	11		
9	12		
10	12		
-----			
L2(10) =			

(Make sure that the pairs of generation numbers and calcumite pairs match in each column.)

### Setting up the window

1. Press **[WINDOW]** to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to 1.

```

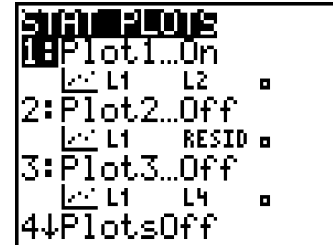
WINDOW
Xmin=0
Xmax=12
Xscl=1
Ymin=0
Ymax=15
Yscl=1
Xres=1
  
```

- Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
- Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to 1.

### Graphing the data: Setting up a scatter plot

In order to analyze the data, you will need to set up a scatter plot and model the data by graphing a line of best fit (logistic regression).

- Press  $\boxed{2\text{nd}} \boxed{[\text{STAT PLOT}]}$  and select **1:Plot1** by pressing  $\boxed{[\text{ENTER}]}$ .

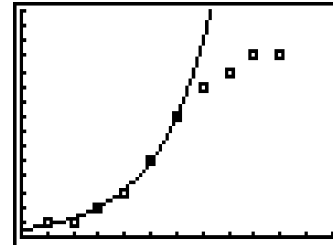


- Set up the plot as shown by pressing  $\boxed{[\text{ENTER}]} \boxed{\downarrow} \boxed{[\text{ENTER}]} \boxed{\downarrow} \boxed{2\text{nd}} \boxed{[L1]} \boxed{[\text{ENTER}]} \boxed{2\text{nd}} \boxed{[L2]} \boxed{[\text{ENTER}]} \boxed{[\text{ENTER}]}$ .



- Press  $\boxed{[\text{GRAPH}]}$  to see the plot.

**Note:** The exponential regression for your ideal population plot is still on the screen. You will use that regression for comparison purposes.





4. The graph to the right shows the rate of calcumite population growth for three intervals of the plot shown in number 3. This is also shown in number 4 in the first part of this activity.

2 to 3 generations

5 to 6 generations

8 to 9 generations

Determine the growth rate for each of those time intervals by determining the *slope* of each of the plots, using the following formula for slope:

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

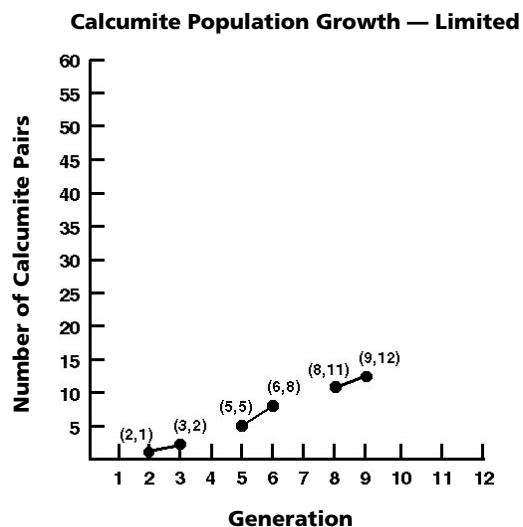
Enter the slopes on the **Data Collection and Analysis** page.

Answer Part II questions 1 and 2 on the **Data Collection and Analysis** page.

It is necessary to determine an appropriate regression model for this data. You know it is not linear since the slope changes. An exponential regression would not be appropriate, since the slope does not continually increase over the three intervals. What type of regression would be a suitable model for this data?

### Analyzing the data: Finding a logistic regression

- Press **STAT** and move the cursor to the **CALC** menu.



```

EDIT [ ] [ ] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg

```

2. Press the up arrow  $\uparrow$  or the down arrow  $\downarrow$  to select **B:Logistic** and press  $\text{ENTER}$ .

```

EDIT 0:00 TESTS
7:QuartReg
8:LinReg(a+bx)
9:LnReg
0:ExpReg
A:PwrReg
B:Logistic
C:SinReg
    
```

```

Logistic
    
```

3. Press  $\text{2nd}$   $[L1]$   $,$   $\text{2nd}$   $[L2]$   $,$ .

```

Logistic L1,L2,
    
```

4. Press  $\text{VAR}$  and move the cursor to the **Y-VARS** menu.

```

VARS 0:0:0:0
1:Function...
2:Parametric...
3:Polar...
4:On/Off...
    
```

5. Select **1:Function** by pressing  $\text{ENTER}$ .

```

FUNCTION
1:Y1
2:Y2
3:Y3
4:Y4
5:Y5
6:Y6
7:Y7
    
```

6. Select **2:Y2** and press  $\text{ENTER}$ .

**Note:** In this step, you are pasting the function in **Y2**, not **Y1**. You want to save the function in **Y1** for comparison purposes.

```

FUNCTION
1:Y1
2:Y2
3:Y3
4:Y4
5:Y5
6:Y6
7:Y7
    
```

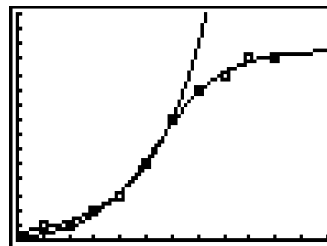
```

Logistic L1,L2,Y
2
    
```

7. Press **ENTER** to calculate the logistic regression and paste the function in **Y2**.

```
Logistic
y=c/(1+ae^(-bx))
a=61.62551145
b=.7667883962
c=12.5766262
```

8. Press **GRAPH** to see the logistic regression.



The logistic equation, which models population growth, is complex beyond the scope of your studies this year. You can, however, make a comparison between a population growing exponentially and one that is growing logistically.

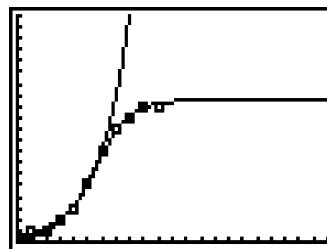
It becomes obvious that the rate at which the population grows is changing; it starts slow, speeds up, and then slows down. This is different from what you observed with ideal population growth (exponential), compared to *limited* population growth (logistic). In order to examine how large the population will become after 20 generations, it is necessary to expand the window.

9. Press **WINDOW** and set the window values as shown.

```
WINDOW
Xmin=0
Xmax=22
Xscl=1
Ymin=0
Ymax=20
Yscl=1
Xres=1
```

10. Press **GRAPH** to see the enlarged generation growth graph.

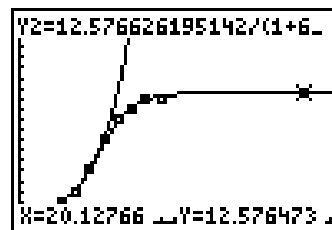
**Note:** This plot levels off. The population size where it levels off is referred to as the carrying capacity.



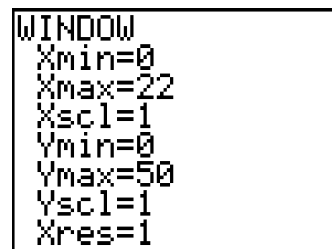
11. To find the carrying capacity, press **TRACE** **▲** so that the cursor is on the **Y2** logistic function.

```
Y2=12.576626195142/(1+6.
X=11 _____ Y=12.410508
```

12. Move the cursor to the right along the level, upper part of the curve until you reach approximately 20 generations ( $x = 20$ ). The  $y$  value is the number of calcumite pairs after 20 generations.



13. Record the approximate size of the population after 20 generations on the **Data Collection and Analysis** page.
14. To get an overview of how the two growth curves compare, reset the window of the calculator as shown.



15. Press **GRAPH** to see the plot.



Answer Part II question 3 on the **Data Collection and Analysis** page.

# Data Collection and Analysis

Name \_\_\_\_\_

Date \_\_\_\_\_

## Activity 4: The Calcumites are Coming!

### Collecting the data

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5					

Ideal Population Growth — Exponential

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	8	10	11	12	12

Limited Population Growth — Logistic

#### Ideal Population Growth

Slope (Interval: 2 – 3 Generations) = \_\_\_\_\_

Slope (Interval: 5 – 6 Generations) = \_\_\_\_\_

Slope (Interval: 8 – 9 Generations) = \_\_\_\_\_

Number of Calcumites  
after 20 Generations \_\_\_\_\_

#### Limited Population Growth

Slope (Interval: 2 – 3 Generations) = \_\_\_\_\_

Slope (Interval: 5 – 6 Generations) = \_\_\_\_\_

Slope (Interval: 8 – 9 Generations) = \_\_\_\_\_

Number of Calcumites  
after 20 Generations \_\_\_\_\_

### Analyzing the data — Part I

1. What does the *slope* of a line on a population plot tell you?

\_\_\_\_\_

\_\_\_\_\_

2. Using the *slope* data, examine and describe the 2 – 3, 5 – 6, and 8 – 9 generation time intervals for the plot of *ideal* growth.

\_\_\_\_\_

\_\_\_\_\_

### Analyzing the data — Part II

1. Describe the same generation time intervals for the plot of *limited* growth.

\_\_\_\_\_

\_\_\_\_\_

2. Which do you think is a more realistic model in nature? Why?

---

---

3. Which model — exponential or logistic — would be the better model to describe the following situations? Explain your answer in each case.

- a. The velocity (speed) of a car going down an endless hill. (Graph time versus speed)

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- b. A virus spreading in your classroom. (Graph time versus number of people infected)

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- c. The number of TI-83 Plus calculators sold. (Graph time versus number sold)

---

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### **Extensions**

- ◆ Leonardo Fibonacci was an Italian mathematician who lived from approximately 1175 – 1250. He is famous for discovering an interesting relationship between sequences of numbers that is now known as the *Fibonacci sequence*. The sequence starts with the following series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and so on. Starting with the third number, each number is equal to the sum of the two numbers preceding it. To study this interesting mathematical phenomenon, divide each number in the sequence by the number before it.

Use your knowledge of the natural world to find other species that contain examples of the Fibonacci sequence. Visualize the sequence by plotting your data. Notice that the data tends to stabilize around a certain number.

## Teacher Notes



### Activity 4

## The Calcumites are Coming!

### Objectives

- ◆ To model the growth of a population
- ◆ To compare ideal population growth with a population whose growth is limited
- ◆ To use technology to find an exponential and logistic regression equation
- ◆ To use technology to plot an exponential and logistic model

### Materials

- ◆ TI-83 Plus
- ◆ Calcumite cut outs, pennies, or candy

### Preparation

- ◆ The calcumite population examined in this activity is similar to the way in which Fibonacci examined a hypothetical rabbit population.
- ◆ Activity 8 will further examine Fibonacci sequences as applied to the spiral of a snail shell.
- ◆ While the exponential regression used to generate the plot of ideal population growth should be discussed, the logistic equation is beyond the scope of an introductory Algebra class. Yet, the general nature of the plot, with its changing slopes, can be studied to give the students an idea of how a population in nature often grows and stabilizes around the carrying capacity.
- ◆ It may be easier for some students to use manipulatives to better visualize the reproductive patterns of the calcumites. Here are a few suggestions:
  - ◆ Make copies of the calcumites on the final page of this activity.
  - ◆ Use coins or candy to simulate the calcumite populations.

## Answers to Data Collection and Analysis questions

### Collecting the data

- ◆ Sample data:

<b>Generation</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>Pair Number</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>5</b>	<b>8</b>	<b>13</b>	<b>21</b>	<b>34</b>	<b>55</b>

**Ideal Population Growth — Exponential**

<b>Generation</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>Pair Number</b>	<b>1</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>5</b>	<b>8</b>	<b>10</b>	<b>11</b>	<b>12</b>	<b>12</b>

**Limited Population Growth — Logistic**

#### Ideal Population Growth

Slope (Interval: 2 – 3 Generations) = 1

Slope (Interval: 5 – 6 Generations) = 3

Slope (Interval: 8 – 9 Generations) = 13

Number of Calcumites  
after 20 Generations 5780

#### Limited Population Growth

Slope (Interval: 2 – 3 Generations) = 1

Slope (Interval: 5 – 6 Generations) = 3

Slope (Interval: 8 – 9 Generations) = 1

Number of Calcumites  
after 20 Generations 12

### Analyzing the data — Part I

1. What does the *slope* of a line on a population plot tell you?

*The slope of a line on a population plot describes the rate of population growth.*

2. Using the *slope* data, examine and describe the 2 – 3, 5 – 6, and 8 – 9 generation time intervals for the plot of *ideal* growth.

*For ideal growth, the rate of growth increases. Observe that the slope increases from 1 to 3 to 13 for the ranges examined.*

### Analyzing the data — Part II

1. Describe the same generation time intervals for the plot of *limited* growth.

*For limited growth, the rate of growth starts slow, speeds up, and then slows down. Observe that the slopes are 1, 3, and 1 for the ranges examined.*

2. Which do you think is a more realistic model in nature? Why?

*The logistic model is more realistic because a population cannot increase in size, at a faster rate, forever. Limiting factors such as food availability, climate, disease, predation, and competition will eventually limit the size of any population.*



3. Which model — exponential or logistic — would be the better model to describe the following situations? Explain your answer in each case.

- a. The velocity (speed) of a car going down an endless hill. (Graph time versus speed)

*Exponential — It will continuously accelerate down this hypothetically endless hill.*

- b. A virus spreading in your classroom. (Graph time versus number of people infected)

*Logistic — It will spread slowly at first as there are only a few infected individuals to pass it on. As more people are infected, they will quickly pass it on. As the number of uninfected individuals lessens, the likelihood of an infected individual contacting an uninfected individual decreases, and so the rate of spread slows down.*

- c. The number of TI-83 Plus calculators sold. (Graph time versus number sold)

*Logistic — The number of people purchasing the TI-83 Plus calculators starts out slow, increases as more people spread the good news, and slows down as few people are left who have not yet purchased them.*

**CALCUMITE CUT OUTS**