

Introduction to Parabolas

Student

7 8 9 10 11 12



Introduction

Parabolas belong to a group of curves called “conic” sections. They are so named because they can be obtained by slicing a cone at various angles. Properties of a parabola make them particularly useful in the world around us particularly in satellite dishes, car headlights and other objects where bringing waves to a single focus or projecting them from a point is useful. Another conic shape is the ellipse, often loosely referred to as an oval. Aside from its obvious sporting applications, the ellipse is used in medical equipment and also describes the motion of planets.

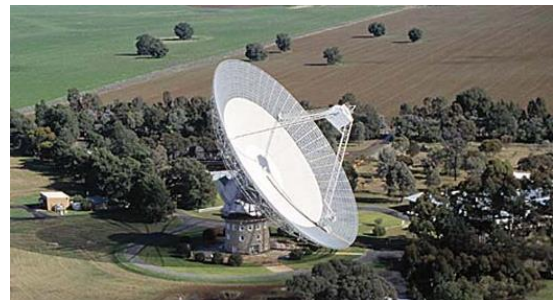


Figure 1 - Parkes Telescope [Courtesy CSIRO]

Creating an envelope

Open the TI-Nspire document: “More than a Graph”

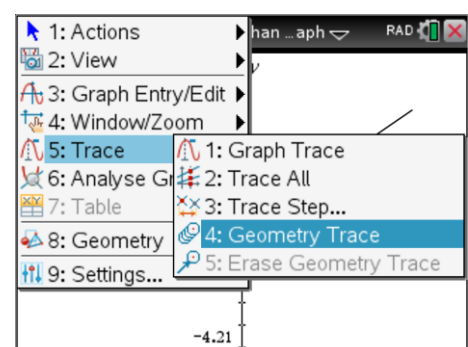
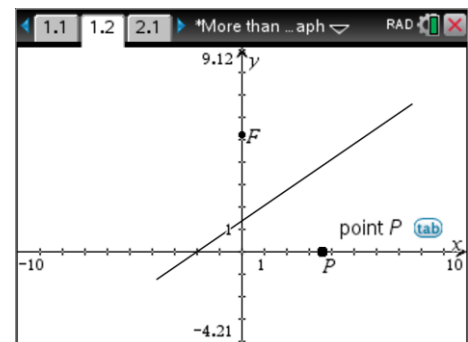
Read the instructions on Page 1.1 and then navigate to Page 1.2 where a geometric construction has been placed on a graph.

Use the mouse over point P then click and hold to grab the point. Move point P along the x axis and watch the path of the line. The line is a ‘perpendicular bisector’. Each point on the line is same distance from P as it is from F (equidistant).

Imagine if the line left a residue to show where it has been. This can be done two ways.

[Menu] > Trace > Geometry Trace

Click on the perpendicular bisector (line) then grab point P and move it along the x axis.



Question: 1

The series of lines left by the Geometry Trace forms an ‘envelope’. What is the shape of the envelope?

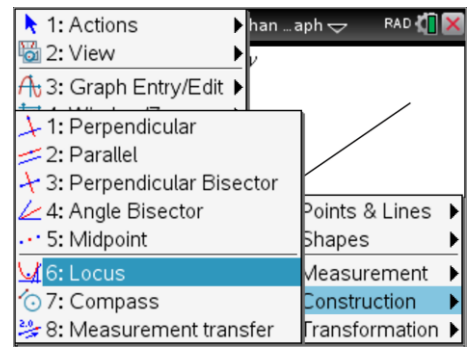
The geometry trace can be cleared off by using the “Erase Geometry Trace” option.

[Menu] > Trace > Erase Geometry Trace

A dynamic trace can be constructed using the Locus tool.

[Menu] > Geometry > Construct > Locus

Click on the line followed by Point P.



Question: 2

Once the Locus has been constructed, grab point F and move it up and down the y axis. How does moving point F change the shape of the envelope?

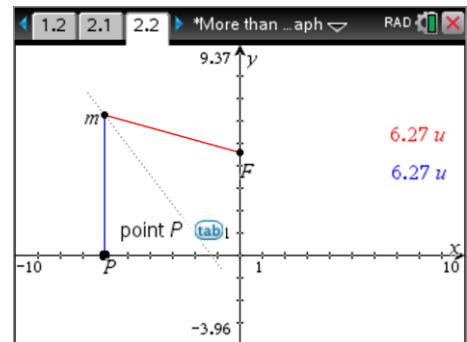
Navigate to page 2.2. Grab point P and observe the position of point M and its distance from P and F.

The dotted line is a perpendicular bisector of points P and F.

A locus can be constructed on point M as point P moves along the x axis.

[Menu] > Geometry > Construct > Locus

Select point M and then point P to see the locus.



Question: 3

What shape is the locus?

Question: 4

If point F is at (0, 4) where would the locus cross the y axis?

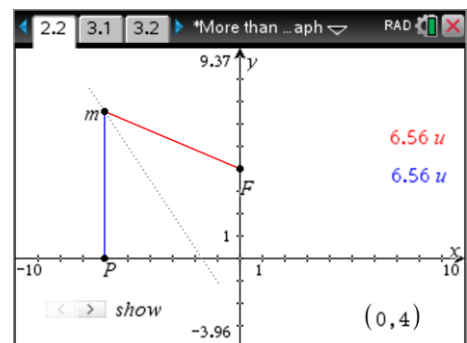
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Select point M and then point P to see the locus.

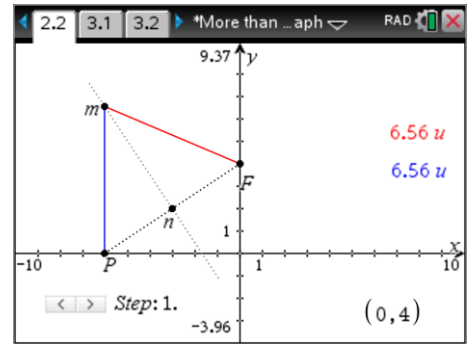


Question: 5

What shape is the locus?

The slider in the bottom left corner of the screen can be used to reveal how the perpendicular bisector helps construct the shape of the locus.

Click on the slider to see Steps 1 through to 3.



Question: 6

Given that n is the midpoint of PF , describe the relationship between $\triangle MNP$ and $\triangle MNF$ and therefore the relationship between lengths MP and MF .

Hint: The slider can help reveal the details.

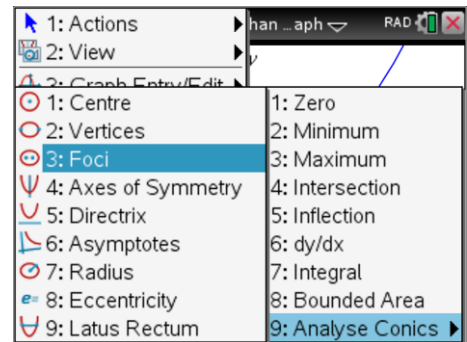
Navigate to page 3.2 where a parabola and its corresponding equation are displayed. This time the calculator will do the geometry.

[Menu] > Analyse Graph > Analyse Conics > Foci

Select the parabola and the 'focus' and its coordinates appear.

[Menu] > Analyse Graph > Analyse Conics > Directrix

Select the parabola and a line called the Directrix will appear.



The focus equates to point F in the previous diagrams. The x axis was used as the directrix in previous diagrams.

Question: 7

Compare the distance from the focus to a point P on the parabola and the distance from P to the directrix.

Application

The radio telescope at Parkes has a 'diameter of 64metres. When radio waves bounce off the surface they are reflected back towards the focal point 27metres away. Imagine the surface of the dish as a two dimensional space.

Question: 8

Determine the area of this space in square centimetres (cm^2).

Question: 9

Suppose the dish is accurate enough to focus these waves into a region that is only 10cm^2 . The energy from the waves collected by the entire dish now fit into this region. Calculate the increase in intensity.

(Intensity: Overall area of dish divided by focused area.)