EXPLORATIONS

Activity 8

At a Snail's Pace

Objectives

♦ To plot a mathematical relationship that defines a spiral

♦ To use technology to create a spiral similar to that found in a snail

♦ To use technology to plot a set of ordered pairs

Materials

♦ TI-83 Plus

♦ Lead pencil

♦ Colored pencils - red and green

♦ Metric ruler

♦ Compass

♦ Tracing paper and graph paper

Introduction

It is hard to imagine a way to mathematically describe a spiral. Spirals are commonly seen in nature. There are a number of different types of spirals. Some interesting examples include the spiral arrangements of scales in pine cones, seeds in a sunflower, or the calcified layers of a snail's shell. One way of constructing a spiral is to use the Fibonacci sequence. This sequence was examined in *Activity 4: The Calcumites are Coming!* The sequence starts with the following series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 and so on. Starting with the third number, each number is equal to the sum of the two numbers preceding it. This sequence can be applied to constructing a spiral resembling a snail's shell.

Problem

A snail shell is composed of a spiral of calcified layers that get wider with each turn. How can this spiral be mathematically modeled?

Collecting the data

1. Place a piece of graph paper on your desk. Draw a set of *x*- and *y*-axes to include the four extreme points (0, 0), (34, 0), (0, 21), and (34, 21).

2. Draw a square by connecting the vertices (10, 5), (10, 6), (9, 6), and (9, 5). Lightly shade this square with a pencil.

3. Construct a square to the left of the original square by connecting vertices (8, 5), (8, 6), (9, 6), and (9, 5). Observe that the two squares together form a rectangle whose width is 1 and whose length is 2.

4. Construct a square above this rectangle by connecting the vertices (10, 6), (10, 8), (8, 8), and (8, 6).

5. Continue to add squares to the graph in a clockwise fashion (see diagram at right) connecting the vertices shown in the table below. The two rows are the 1 X 1 and 2 X 2 squares that you already constructed. The last row is a 21 X 21 square.
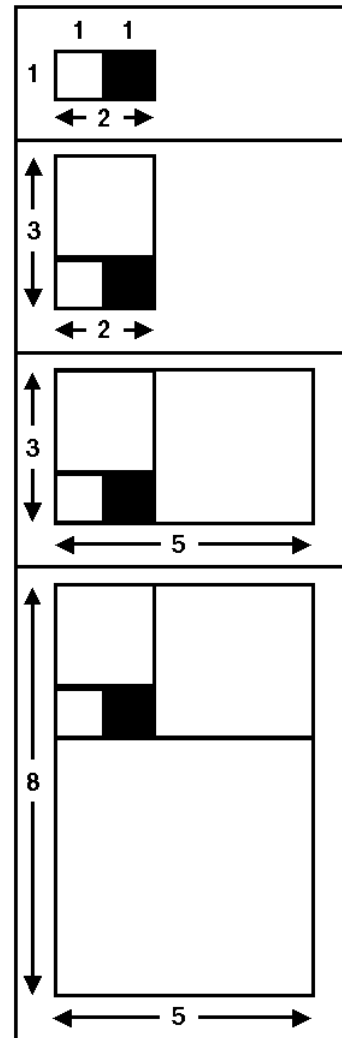
Length of side of square	Vertices			
1	(8, 5)	(8, 6)	(9, 5)	(9, 6)
2	(8, 6)	(8, 8)	(10, 6)	(10, 8)
3	(10, 5)	(10, 8)	(13, 5)	(13, 8)
5	(8, 0)	(8, 5)	(13, 0)	(13, 5)
8	(0, 0)	(0, 8)	(8, 0)	(8, 8)
13	(0, 8)	(0, 21)	(13, 8)	(13, 21)
21	(13, 0)	(13, 21)	(0, 34)	(34, 21)

6. Each time a square is added a new rectangle is formed. Record the dimensions of each rectangle on the **Data Collection and Analysis** page.

Answer questions 1 and 2 on the **Data Collection and Analysis** page.

7. Using a red pencil, mark each of the points shown in the table below on your graph paper. These points are each one of the corners of the squares in the graph. The dimensions of the sides of each square are also given in the table.

Point	(9, 6)	(10, 6)	(10, 5)	(8, 5)	(8, 8)	(13, 8)	(13, 0)	(0, 0)
Dimension of sides	1	2	3	5	8	13	21	34

8. Using a compass, construct a ¼ circle in the square that measures 1 X 1. Place the point of the compass on the red point at vertices (9, 6) that you made in step 7. Open up the compass so that the pencil is on an adjacent vertex. Sweep a ¼ circle or arc that goes to the opposite corner of the square.

9. Repeat this step with each of the squares, using the red marked points as the place to position the compass point. Observe that you are constructing a continuous line that resembles a spiral — a Fibonacci spiral. The result of the first three squares is shown to the right.
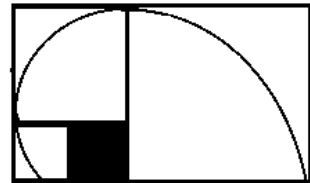
Setting up the TI-83 Plus

Before starting your data collection, make sure that the TI-83 Plus has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Setting up the window

1. Press WINDOW to set up the proper scale for the axes.

2. Set the **Xmin** value by identifying the minimum value. Choose a number that is less than the minimum.

3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. Set the **Xscl** to **0**.

4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.

5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **0**.


```
WINDOW
 Xmin=-35
 Xmax=35
 Xscl=0
 Ymin=-35
 Ymax=35
 Yscl=0
 Xres=1
```

Entering the data in the TI-83 Plus

Construct a Fibonacci spiral using the TI-83 Plus. You can construct circles using the same center points and radii that were used in the previous steps. Although you must draw full circles, you are only interested in one-fourth of each circle. Collectively, the circle segments will form the Fibonacci spiral.

1. Press [2nd] [QUIT] to return to the Home screen.

2. The first circle has a center at point (9, 6) and a radius of 1. Press [2nd] [DRAW] to draw the circle.

3. Press the up arrow [▲] or the down arrow [▼] to select **9:Circle(** and press [ENTER].

4. Enter **9** [,] **6** [,] **1** [)] and press [ENTER].

    A small circle is drawn as shown.
   The center of the circle is (9, 6) and the radius is 1.
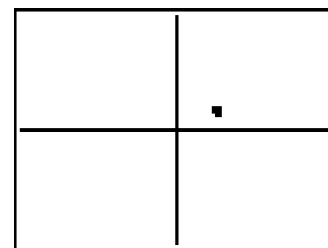
5. Repeat steps 1 through 4 for each circle. Change the center point and radius according to the table below each time step 4 is repeated.

| Center point	Radius	Home screen shows
(10, 6)	2	Circle(10,6,2)
(10, 5)	3	Circle(10,5,3)
(8, 5)	5	Circle(8,5,5)
(8, 8)	8	Circle(8,8,8)
(13, 8)	13	Circle(13,8,13)
(13, 0)	21	Circle(13,0,21)
(0, 0)	34	Circle(0,0,34)

Where is the spiral on the diagram shown at the right?

The first circle that you constructed appears as a dot in the first quadrant of the graph. Remember that you only need one-fourth of each circle that was drawn.

In the diagram shown at the right, three-fourths of each circle that was constructed is erased, leaving a spiral. This action cannot be done on the TI-83 Plus.
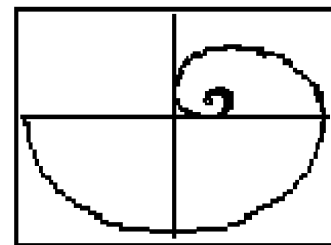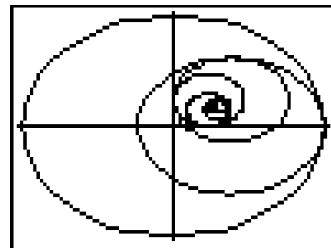
6. Use tracing paper to help spot the spiral on the TI-83 Plus. Using a pencil, draw a set of axes on the tracing paper. Place the paper over the TI-83 Plus screen so that the axes on the paper align with the axes on the screen. Using a red pencil, find and trace the spiral by finding the dark center of the spiral and proceeding clockwise around it.

7. Use the first screen in number 5 as a guideline. You should now have a spiral. How does this spiral compare with the spiral that you constructed on the graph paper?

8. Save the drawing. Press [2nd] [DRAW] and move the cursor to the **STO** menu.
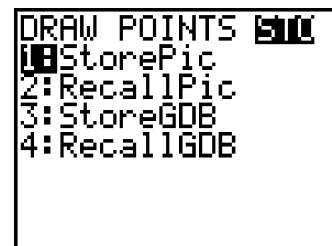
9. Select **1:StorePic** by pressing ⌈ENTER⌉.

```
StorePic
```

10. Press ⌈VARS⌉.

```
VARS Y-VARS
1:Window…
2:Zoom…
3:GDB…
4:Picture…
5:Statistics…
6:Table…
7:String…
```

11. Select **4:Picture** and press ⌈ENTER⌉.

```
VARS Y-VARS
1:Window…
2:Zoom…
3:GDB…
4:Picture…
5:Statistics…
6:Table…
7:String…
```

12. Select **1:Pic1** by pressing ⌈ENTER⌉. The picture (including the window settings) is saved for future use.

```
PICTURE
1:Pic1
2:Pic2
3:Pic3
4:Pic4
5:Pic5
6:Pic6
7↓Pic7
```

```
StorePic Pic1
```

Data Collection and Analysis

Name _____

Date _____

Activity 8: At a Snail's Pace

Collecting the data

Dimensions of Rectangles	
Width	**Length**
1	2

Analyzing the data

1. Each square was added in a clockwise direction around the original shaded square. How many squares have to be added to the plot in order to draw one complete turn of the spiral?

_____

2. The sequence of widths and lengths in the table above are part of a Fibonacci sequence. Determine the next five widths and lengths. Add those to the table below.

Width	**Length**

3. Give examples of Fibonacci sequences from this activity.

_____

_____

Extensions

♦ How much do the Fibonacci rectangles increase in area each time a square is added on?

Look at how the areas of the rectangles change as you build larger and larger rectangles. Find out by what factor the area changes each time you increase the size of the rectangles. Two formulas can be used:

L - S = ΔA

ΔA ÷ S = Increase Factor

where *L* is the area of the large rectangle, *S* is the area of the small rectangle, and ΔA is the change in area.

Is there a golden lesson in this activity? Does the graph generated in this exercise support your ideas about a Golden Ratio?

Note: Activity 4: The Calcumites are Coming! and this activity both dealt with the Golden Ratio of 1.618. This ratio is obtained by dividing the numbers in a Fibonacci sequence by the preceding values in the sequence. Observe that the area of the rectangles increases by a factor of 1.618. When using a spiral of squares to construct a spiral each quarter turn of the spiral increased in distance from the center by a factor of 1.618. In a Fibonacci spiral, the distance from the center increases by a factor of 1.618 with each complete turn of the spiral.

Teacher Notes

A c t i v i t y 8

At a Snail's Pace

♦ To plot a mathematical relationship that defines a spiral

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Preparation

♦ For the first part of this activity it is important to select appropriate graph paper that allows students to visualize the spiral of squares and the spiral that they generate when drawing the arcs within the squares.

♦ Fibonacci sequences can be seen throughout this activity: 1) sides of the squares; 2) widths of the rectangles; 3) lengths of the rectangles; and 4) radii of the concentric circles that form the spiral.

♦ Golden Ratios are embedded within this activity, although only one is calculated. The one that the students calculated is the increase in the area of the Fibonacci rectangles. Other examples include: 1) the increase in distance from the center to subsequent turns in a Fibonacci spiral, and 2) the length of the rectangles divided by the width of the rectangles.

♦ The Fibonacci sequence is not the only sequence that approaches the Golden Ratio. Students can build a sequence by starting with any two numbers where, beginning with the third number, each number is equal to the sum of the preceding two numbers: 3, 7, 10, 17, 27, 44, 71, 115, and so on. The Golden Ratio is obtained by dividing each number by the preceding number in the sequence.

Answers to Data Collection and Analysis questions

Collecting the data

◆ Sample data:

| Dimensions of Rectangles ||
Width	Length
1	2
2	3
3	5
5	8
8	13
13	21
21	34
34	55
55	89

Analyzing the data

1. Each square was added in a clockwise direction around the original shaded square. How many squares have to be added to the plot in order to draw one complete turn of the spiral?

   Four.

2. The sequence of widths and lengths in the table above are part of a Fibonacci sequence. Determine the next five widths and lengths. Add those to the table below.

   See chart.

Width	Length
89	144
144	233
233	377
377	610
610	987

3. Give examples of Fibonacci sequences from this activity.

   See the Teacher Notes from Activity 4.