Activity Overview: *Students will be given piecewise functions and asked to evaluate both the left-hand limit and the right-hand limit of the function as x approaches a given number, c. Using sliders, students will estimate the value of the missing variable that makes the left-hand limit and the right-hand limit equal.*

Topic: Limits

- *One Sided Limits*

Teacher Preparation and Notes

- *Students should already have been introduced to one-sided limits. They should also know how to evaluate a one-sided limit graphically.*
- *Students should know that a limit exists if and only if the left-hand limit and the right-hand limit are equal.*
- *Estimated time = 20 minutes*

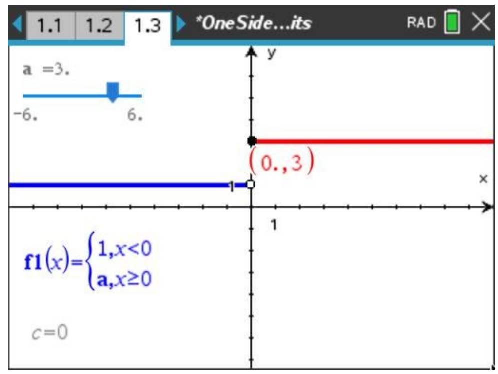
Associated Materials

- *OneSidedLimits_Student.doc*
- *OneSidedLimits.tns*

Students will read and follow the directions on page 1.2. For Problems 2 and 3, students are asked whether the function table of values is consistent or inconsistent with the value of a that ensures that the limit exits, and to find the value of a algebraically.

Problem 1

On page 1.3, before moving the slider, students will graphically estimate the limit of **f1**(x) as x approaches 0 from the left and the right. Students will then use the slider to graphically estimate the value of a that will ensure that the limit of **f1**(x) as x approaches zero exists.



Student Worksheet solutions

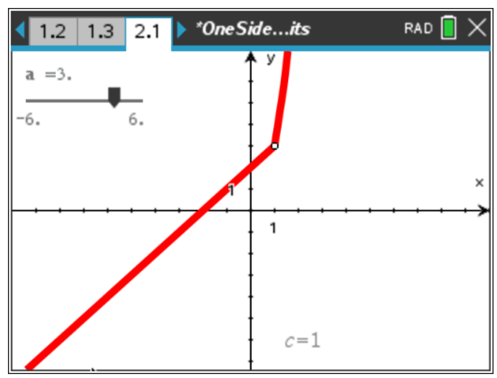
1. $\lim\limits_{x\to 0^-} \mathbf{f1}(x) \approx 1$

2. $\lim\limits_{x\to 0^+} \mathbf{f1}(x) \approx 5$

3. $a \approx 1$

Problem 2

On page 2.1, before moving the slider, students will graphically estimate the limit of **f1**(x) as x approaches 1 from the left and the right. Students will then use the slider to graphically estimate the value of a that will ensure that the limit of **f1**(x) as x approaches one exists.



Student Worksheet solutions

1. $\lim\limits_{x\to 1^-} \mathbf{f1}(x) \approx 3$

2. $\lim\limits_{x\to 1^+} \mathbf{f1}(x) \approx 5$

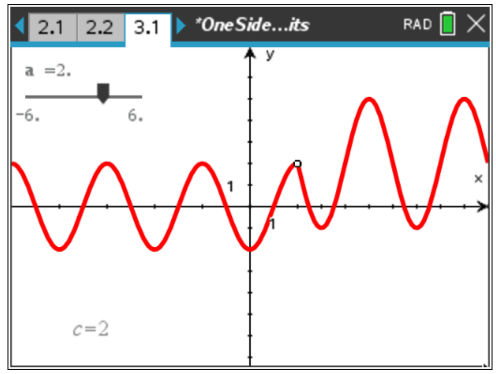
3. $a \approx 3$

4. Consistent

5. $1 + 2 = a \cdot 1^2$; $a = 3$

Problem 3

On page 3.1, before moving the slider, students will graphically estimate the limit of **f1**(*x*) as *x* approaches 2 from the left and the right. Students will then use the slider to graphically estimate the value of *a* that will ensure that the limit of **f1**(*x*) as *x* approaches two exists.



Student Worksheet Solutions

1. $\lim\limits_{x \to 2^-} \mathbf{f1}(x) \approx 2$

2. $\lim\limits_{x \to 2^+} \mathbf{f1}(x) \approx 5$

3. $a \approx 2$

4. Consistent

5. $$2\sin\left(\frac{\pi}{2}(2-1)\right) = 3\sin\left(\frac{\pi}{2}(2-4)\right) + a$$
$$2\sin\left(\frac{\pi}{2}\right) = 3\sin(-\pi) + a$$
$$2 \cdot 1 = 3 \cdot 0 + a$$
$$2 = a$$

Extension – Continuity

Students are introduced to the concept of continuity and are asked if each of the functions in Problems 1–3 is continuous at c given the value of **a** found earlier. For the functions that are not continuous, they are asked how the function can be modified to make it continuous.

Student Worksheet Solutions

1. Continuous because all of the *x*-values in the neighborhood of $x = 0$ are included in the domain of the function.

2. Not continuous because $x = 1$ is not included in the domain of the function. To make the function continuous at $x = 1$, either change the interval in the first branch to $x \leq 1$ or change the interval in the second branch to $x \geq 1$.

3. Not continuous because $x = 2$ is not included in the domain of the function. To make the function continuous at $x = 2$, either change the interval in the first branch to $x \leq 2$ or change the interval in the second branch to $x \geq 2$.