EXPLORATIONS

Activity 7

What Goes Up Must Come Down

Teacher Notes

Concepts

- Solving simple quadratic equations
- Using the quadratic formula to solve problems

Calculator Skills

- Using the square root: [2nd] [√‾]
- Using [2nd] [ANS] and [MEMVAR] to solve and check problems

Materials

- TI-30X IIS
- Student Activity pages (p.69-72)

Objective

- In this activity, students will use the calculator to solve simple quadratic equations. They also will use the quadratic formula to determine the vertex and x-intercepts of the graph of a quadratic function.

Topics Covered

- Representing situations that involve variable quantities with expressions and equations
- Using tables and graphs as tools to interpret expressions and functions
- Modeling real-world phenomena with the quadratic formula

Introduction

Archeologists can learn much about ancient cultures by examining their art forms. Many art forms include symmetry as a basic feature. Monarch butterflies exhibit symmetry when their wings are spread. In mathematics, a parabola is an example of symmetry in a graph. A function or an equation whose graph is a parabola is called a *quadratic function.* The word quadratic comes from a Latin word that means *to make a square*. How can you sketch the graph of a parabola simply by observing the algebraic representation of the quadratic function that describes it?

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Investigation

1. Solve the following quadratic equation.

   $\frac{1}{3}b^2 - 6 = 4$

   Perform symbolic manipulations to get:

   $\frac{1}{3}b^2 = 10$ $b^2 = 30$

2. Use the overhead calculator to find the square root of 30 two ways — $\sqrt{30}$ or $30^{1/2}$. This is the positive solution.

Press:	**The calculator shows:**
[CLEAR] [2nd] [√‾] 30 [)] [ENTER]	√ (30) 5.477225575 DEG
30 [^] [(] 1 [÷] 2 [)] [ENTER]	30^(1/2) 5.477225575 DEG

   To check the calculated value for *b,* store this value in variable B. Then use the value in the original equation.

   $\frac{1}{3}b^2 - 6 = 4$

Press:	**The calculator shows:**
[STO▸] [▸] [ENTER]	Ans → B 5.477225575 DEG
[(] 1 [÷] 3 [)] [MEMVAR] [▸] [ENTER] [x^2] [−] 6 [ENTER]	(1/3)B² − 6 4 DEG

3. Remember that the solution of the original equation yields both a *positive* and a *negative* value for *b*. Try storing the *negative* value for *b* and check to confirm the solution.

Wrap-Up

Use student-prepared graphs of data from real-world phenomenon that clearly show the vertical intercept (axis of symmetry), and the x-intercepts.

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Display the graphs, and have students explain the possible meaning of the intercepts and the vertex.

Extensions

- Refer to the baseball problem in Student Activity Part 2. Use guess-and-check to find the time *t* when the ball will be at least 25 feet above the ground.
- What is the meaning of the y-intercept in the graph from Student Activity Part 3?
- Answer the original question posed in the Introduction.

Solutions Part 1

Solve and check each equation by using a combination of symbolic techniques and your calculator. Be sure to indicate *both* positive and negative solutions.

1.	$5B^2 - 90 = 0$	*±4.2426*	6.	$10 = {}^-12E^2 - 5$	*±1.1180*
2.	$r^2 - 0.64 = 0$	*±0.8*	7.	$\frac{3}{4}x^2 - 9 = 47$	*±8.6410*
3.	${}^-2p^2 + 3 = -21$	*±3.4641*	8.	$8 + \frac{3}{7}x^2 = 22$	*±5.7155*
4.	$\frac{1}{2}n^2 - 5 = 39$	*±9.3808*	9.	$90 = \frac{5}{8}x^2 - 18$	*±13.1453*
5.	$20 - 4t^2 = 9$	*±1.6583*	10.	$5.8 + 6.2B^2 = 3.7B^2 + 7.9$	*±0.9165*

11. Agarwal is sliding down a giant slide at an amusement park. Agarwal's height *h* (in feet) above the ground at any given point in time is given by $h(t) = 75 - 8t^2$, where *t* is the time (in seconds) after he begins to start down the slide.

    A. How tall is the slide?

    75 feet

    B. After how many seconds will Agarwal be 20 feet above the ground on his descent?

    t = 2.62 seconds

    C. After how many seconds will Agarwal reach the bottom of the slide if the slide is 2.5 feet off the ground?

    t = 3.0104 seconds

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Solutions Part 2

Jeremy throws a baseball straight up into the air with a velocity of 46 feet per second as it leaves his hand (at 6 feet above the ground). The function $h(t) = {}^{-}16t_2 + v_0t + h_0$ is a model used by scientists for the height of a projectile, in feet, as a function of time, in seconds. In this model v_0 is the initial velocity of the projectile and h_0 is the initial height from which the projectile is thrown or dropped. The model is not perfect, since it does not take into account the factor of air resistance. Air resistance can affect the projectile's vertical movement. For our purposes, however, we will assume that the effect of air resistance is negligible.

1. Use the function and your calculator to complete the (*time, height*) table.

   Enter the function for the height of the baseball (the projectile). Use the variable 'A' to represent the variable *t* in the equation. Enter the function as ${}^{-}16A^2 + 46A + 6$. To evaluate the expression for any time *t*, first store the value in A.

TIME (*seconds*)	HEIGHT (*feet*)
0	*6*
0.5	*25*
1.0	*36*
1.5	*39*
2.0	34
2.5	*21*
3.0	*0*
3.5	*-29*

2. On a separate sheet of paper, graph the function on the coordinate grid.

3. According to the table, what is the maximum height for the ball?

   The height is a maximum of 39 feet according to the table.

   How do you know?

4. When does the ball reach that height?

   At t = 1.5 seconds

5. If Jeremy catches the ball when it is 6 feet above the ground, how many seconds has the ball been in the air since it was thrown?

   About 2.875 seconds.

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Solutions Part 3

The quadratic formula can be used to explore problem situations.

Use the following form of the quadratic formula and your calculator to solve the baseball problem in Student Activity Part 2.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

1. Let $x = t$, and let $a = {}^{-}16$, $b = 46$, and $c = 6$. Sketch the axis of symmetry for the quadratic function model on the coordinate grid using $x = \frac{-b}{2a}$. What is this value?

   1.4375

2. Store the value that you just found in variable A, and evaluate the expression as you did in Student Activity Part 2 to find the height for the value at A seconds.

   Compare your result to the maximum height you found in Student Activity Part 2. The ordered pair (A, height at A) that you just found is called the *vertex*. The vertex of a parabola is where the maximum or minimum point occurs. Plot the vertex on the coordinate grid from #1 as the maximum point.

   t = 1.4375 and a maximum height of 39.0625 feet.

3. Calculate $\pm\frac{\sqrt{b^2 - 4ac}}{2a}$ using the same values for *a, b,* and *c*.

   The two values for this expression are *+1.5625 and ⁻1.5625.*

4. Use your calculator to add the two values found in step 3 to the value 1.4375. These values are *⁻0.125* and *3.*

5. Plot these values as the x-intercepts of the graph on the coordinate grid from #1. Sketch a parabola on the grid by connecting the maximum point found in step 2 with the two x-intercepts found in step 4. What is the meaning of the x-intercepts as they relate to the problem situation?

   The x-intercepts represent the times when the ball would be "on the ground." Since the ball was thrown from a height of 6 feet and caught at 6 feet, it will never touch the ground!

6. How is the axis of symmetry related to the x-intercepts?

   The axis of symmetry is the median of the x-intercepts.

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7. Calculate the average value of the x-intercepts. What do you observe about this value?

   The average value of the x-intercepts is 1.4575. That is the value for the axis of symmetry.

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Student Activity 7

Name ____________________

Date ____________________

Quadratic Models—What Goes Up Must Come Down

Objective: *In this activity, you will use the calculator to solve simple quadratic equations. You also will use the quadratic formula to determine the vertex and x-intercepts of the graph of a quadratic function.*

Part 1: Solving Quadratic Equations

Solve and check each equation by using a combination of symbolic techniques and your calculator. Be sure to indicate *both* positive and negative solutions.

1. $5B^2 - 90 = 0$
2. $r^2 - 0.64 = 0$
3. $^-2p^2 + 3 = -21$
4. $\frac{1}{2}n^2 - 5 = 39$
5. $20 - 4t^2 = 9$
6. $10 = {}^-12E^2 - 5$
7. $\frac{3}{4}X^2 - 9 = 47$
8. $8 + \frac{3}{7}X^2 = 22$
9. $90 = \frac{5}{8}X^2 - 18$
10. $5.8 + 6.2B^2 = 3.7B^2 + 7.9$

11. Agarwal is sliding down a giant slide at an amusement park. Agarwal's height *h* (in feet) above the ground at any given point in time is given by $h(t) = 75 - 8t^2$, where *t* is the time (in seconds) after he begins to start down the slide.
    a. How tall is the slide?
    b. After how many seconds will Agarwal be 20 feet above the ground on his descent?
    c. After how many seconds will Agarwal reach the bottom of the slide if the slide is 2.5 feet off the ground?

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Part 2: Solving a Quadratic Model

Jeremy throws a baseball straight up into the air with a velocity of 46 feet per second as it leaves his hand (at 6 feet above the ground). The function $h(t) = {}^{-}16t^2 + v_0t + h_0$ is a model used by scientists for the height of a projectile, in feet, as a function of time, in seconds. In this model v_0 is the initial velocity of the projectile and h_0 is the initial height from which the projectile is thrown or dropped. The model is not perfect, since it does not take into account the factor of air resistance. Air resistance can affect the projectile's vertical movement. For our purposes, however, we will assume that the effect of air resistance is negligible.

1. Use the function and your calculator to complete the (*time, height*) table.

   Enter the function for the height of the baseball (the projectile). Use the variable A to represent the variable *t* in the equation. Enter the function as $^{-}16A^2 + 46A + 6$. To evaluate the expression for any time *t*, first store the value in A.

TIME (*seconds*)	HEIGHT (*feet*)
0	
0.5	
1.0	
1.5	
2.0	34
2.5	
3.0	
3.5	

2. On a separate sheet of graph paper, graph the function on the coordinate grid.

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3. According to the table, what is the maximum height for the ball? How do you know?

4. When does the ball reach that height?

5. If Jeremy catches the ball when it is 6 feet above the ground, how many seconds has the ball been in the air since it was thrown?

Part 3: Graphing Problems with the Quadratic Formula

The quadratic formula can be used to explore problem situations.

Use the following form of the quadratic formula and your calculator to solve the baseball problem in Student Activity Part 2.

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

1. Let $x = t$, and let $a = -16$, $b = 46$, and $c = 6$. Sketch the axis of symmetry for the quadratic function model on separate sheet of graph paper using $x = \frac{-b}{2a}$. What is this value?

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2. Store the value that you just found in variable A and evaluate the expression as you did in Student Activity Part 2 *to* find the height for the value at A seconds.

   Compare your result to the maximum height you found in Student Activity Part 2. The ordered pair (A, height at A) that you just found is called the *vertex*. The vertex of a parabola is where the maximum or minimum point occurs. Plot the vertex on the coordinate grid from #1 as the maximum point.

3. Calculate $\pm\frac{\sqrt{b^2-4ac}}{2a}$ using the same values for *a, b,* and *c*. The two values for this expression are +________________ and -_________________.

4. Use your calculator to add the two values found in step 3 to the value 1.4375. These values are _________________ and _________________.

5. Plot these values as the x-intercepts of the graph on the coordinate grid from #1. Sketch a parabola on the grid by connecting the maximum point found in step 2 with the two x-intercepts found in step 4. What is the meaning of the x-intercepts as they relate to the problem situation?

6. How is the axis of symmetry related to the x-intercepts?

7. Calculate the average value of the x-intercepts. What do you observe about this value?

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