Motion in a straight line where displacement is of the form $x = a\begin{cases}\sin \\ \cos\end{cases}(x)+b$ **.**

Mathematics required:
- sketching trigonometric equations.
- solving trigonometric equations over a given domain.
- knowledge of what differentiation of a functions.
- area under the curve.
- knowledge of displacement, velocity and acceleration of a particle in a straight line.
- drawing displacement-time; velocity-time and acceleration-time graphs, including their interpretation.

Technology required:
- Using the Y= EDITOR to enter equations of graphs.
- Setting the WINDOW.
- Graphing a function and ability to read scale on axes.
- Use of TRACE and TABLE to identify coordinates.
- Finding points of intersection of graphs using **F5,5**.
- Using **Style** and **Draw** commands to modify or label graphs.
- Use of **Solve(** and/or **Zeros(** commands.
- Using the **derivative** command from the **HOME** screen.
- Using the **derivative** command from the Y= EDITOR.
- Find the tangent to a curve at a given point using **Math (F5),A** – tangent.
- Calculate the area under a curve using **Math (F5),7 -** integral function.

Introduction

Part A

Algebraic reasoning must be shown.

Consider the graph of $y = 3\sin x + 1$, where x is measured in radians.

a. State the amplitude, period and vertical shift (y-translation) for this graph.

b. Sketch the graph, for one period, labelling the x and y intercepts and all turning points. (Give answers to two decimal places.)

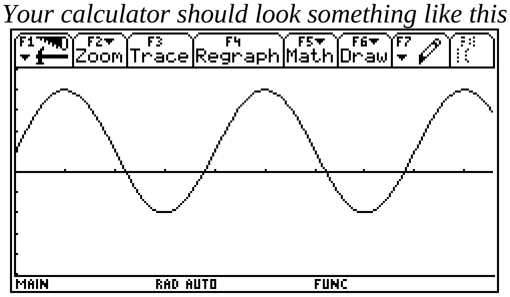
Part B

Technology solutions may be used.

c. Sketch the graph of $y = 3\sin x + 1$ over the domain $0 \le x \le 15$.

Set the Window to x**min=**0; x**max=**15; x**scl=** $\frac{\pi}{2}$; **ymin=** -5; y**max=**5; y**scl=1**.

Your calculator should look something like this

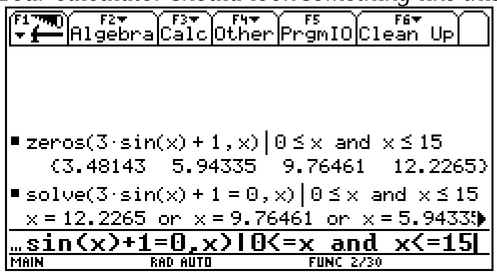


d. Sketch this graph, as shown, and label the scales of the axes, together with the x and y intercepts and turning points.

e. Find the *y* value of the end point where $x = 15$, correct to 2 decimal places.

f. Use **zeros(...** or **solve(...** to confirm *x* and *y* intercepts.
Tip: Use **RADIAN MODE** and ♦**ENTER** for approximate solutions.

Your calculator should look something like this



Tip: *The* ***WITH*** *symbol* (|) *is accessed by* 2nd k.

g. Summarise the characteristics of this graph in a table like this.

equation	graph shape	x intercepts	y intercept	$x = 15, y = ...$	maximum points	minimum points

Application

A particle moves in a straight line so that its displacement, *x* metres, from a fixed point *O*, at time t seconds, is given by $x(t) = 3\sin t + 1, 0 \le t \le 15$.

Use the graph from **Part B** to help answer the following questions.

a. **i.** State the times when the particle is at rest. (Give answers correct to 2 decimal places.)

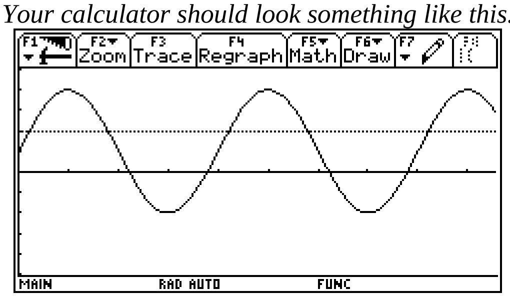
ii. What is the displacement of the particle at these times?

iii. How far has the particle travelled between any two successive times when the velocity is zero?

b. What is the particle's initial position?

c. In the **Y= Editor** add **y2=2** and hence draw graphs for $x(t)$ and $x = 2$ on the same axes.

Your calculator should look something like this.

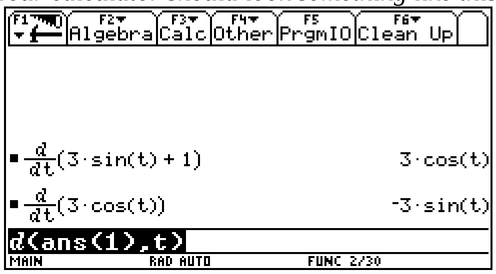


d. Hence, find the times when the particle has a displacement of 2 metres (correct to two decimal places).

e. In the **HOME** screen, use the **derivative** function to find equations for:

- **i.** the velocity of the particle at any time *t*.
- **ii.** the acceleration of the particle at any time *t*.

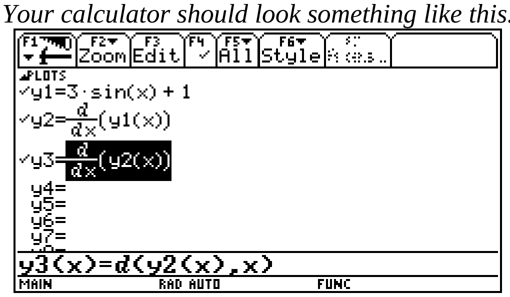
Your calculator should look something like this.



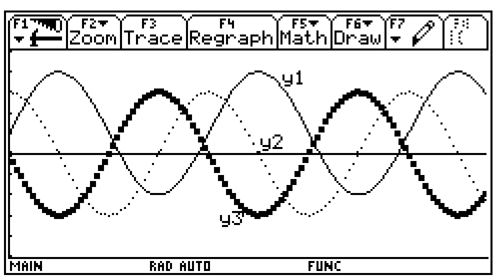
Tip: Use **2nd 8** to access the derivative function ***d***.

f. Sketch the graphs of the displacement-time, velocity-time and acceleration- time on the same set of axes over the domain $x \in [0,15]$. (First on the calculator and then copy to paper, labelling all *x* and *y* intercepts and any maximum and minimum turning points.)

Your calculator should look something like this.



Tip: *Graph each using a different* **Style (F5)**.



Tip: Label each graph using **Style (F6)**.

g. From the velocity-time graph, **i.** state the times when the velocity is zero. **ii.** state the maximum velocity and minimum velocity.

h. From the acceleration-time graph, state the times when the acceleration is zero.

i. On a number line show the motion of the particle for the first fifteen seconds of motion.

j. Give an explanation of the motion of the particle, in particular give attention to:

i. the position of the particle and the direction of the acceleration when the particle is at rest.

ii. the velocity and acceleration when the particle's displacement is 1 ($x = 1$).

This motion is called SIMPLE HARMONIC MOTION

<u>Extensions</u>

a. **i.** Sketch the velocity-time graph: $v = 3\cos t$, $0 \le t \le 15$.

Using **F5,7**. find the area under the curve from $t = 0$ to $t = \frac{\pi}{2}$.

Compare this value to the '*y*' ordinate, for the same *t* value, of the displacement-time graph $x = 3\sin t + 1$, $0 \le t \le 15$.

Repeat this for the following intervals $t \in [0, \pi]$; $t \in \left[0, \frac{3\pi}{2}\right]$; $t \in [0, 2\pi]$; $t \in [0, 15]$.

What relationship does this show between the area under the velocity-time graph and the '*y*' ordinate of the displacement-time graph?

ii. Sketch the velocity-time graph: $v = 3\cos t$, $0 \le t \le 15$.

Using **F5,A**. find the gradient of the tangent to the curve at $t = 0$ and $t = \frac{\pi}{2}$.

Compare these values to the '*y*' ordinate, for the same *t* value, of the acceleration-time graph $a = -3\sin t$, $0 \le t \le 15$.

What relationship does this show between the gradient of the tangent of the velocity-time graph and the '*y*' ordinate of the acceleration-time graph?

b. **i.** Carry out the above investigation on:

$y = 2\sin x - 1$ and $x = 2\sin t - 1$, $0 \le t \le 15$; and

$y = 3\cos x$ and $x = 3\cos t$, $0 \le t \le 15$.

ii. Explain how the graphs of $x = a\sin t + b$ differ from $x = a\cos t + b$.