Chill Out:
How Hot Objects Cool

When you have a hot drink, you know that it gradually cools off. Newton's law of cooling provides us with a model for cooling. It states that the temperature difference T_{diff} between a hot object and its surroundings decreases exponentially with time.

$$T_{diff} = T_0 \, e^{-kt}$$

In the model T_0 is the initial temperature difference, and k is a positive constant.

In this activity you will use a Temperature Probe to collect data as the warmed probe cools. You can then fit several mathematical models to the data.
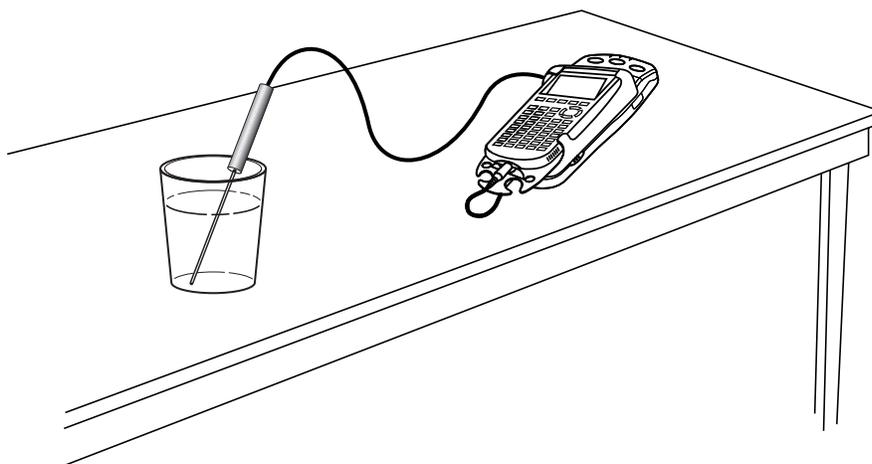
OBJECTIVES

- Record temperature versus time cooling data.
- Model cooling data with an exponential function.

MATERIALS

TI-84 Plus or TI-84 Plus graphing calculator
EasyData application
EasyTemp **or**
   Temperature Probe and data-collection interface
hot water

PROCEDURE

1. Turn on the calculator. Connect the Temperature Probe to the calculator. (This may require the use of a data-collection interface.)

2. Set up EasyData for data collection.

   a. Start the EasyData application, if it is not already running.
   b. Select (File) from the Main screen, and then select **New** to reset the application.

3. Obtain a cup of hot water at 45 to 55 ºC. Place the Temperature Probe in the water, and wait about 20 seconds for the probe to reach the temperature of the water. Rather than waiting for the water to cool, just remove the Temperature Probe from the water and observe the cooling of the probe itself. Remove the probe from the water and rest it on the edge of a table. Do not let anything touch the tip of the probe.

4. Collect the cooling data. Select (Start) from the Main screen. Data collection will run for three minutes, after which a graph of temperature versus time will display.

5. Newton's law of cooling models the temperature difference between the warm object and its surroundings. As an estimate of the room temperature, use the minimum temperature on the graph. Inspect the graph to determine its minimum temperature reading.

   a. Trace along the graph with the ⊙ key and determine the minimum temperature reached. Round this temperature down to the next whole degree (as in 24.54 → 24) and record the value in the Data Table on the *Data Collection and Analysis* sheet.
   b. Select (Main) to return to the Main screen.
   c. Exit EasyData by selecting (Quit) from the Main screen and then selecting (OK).

ANALYSIS

1. Since the model for Newton's law of cooling uses the difference between the temperature of the warm object and its surroundings, subtract the minimum temperature from the measured temperature before comparing data to the model.

   a. Press (2nd) [L2].
   b. Press (−) then key in the (minimum temp), where (minimum temp) is the numerical value you entered in the Data Table.
   c. Press (STO▶), and press (2nd) [L2] a second time. Your expression will look like L2 − 24 → L2, depending on your particular minimum temperature. Press (ENTER) to perform the calculation.

2. Redisplay your graph of temperature difference versus time.

   a. Press (2nd) [STAT PLOT].
   b. Press (ENTER) to select Plot1 and then press (ENTER) again to select On.
   c. Press (ZOOM).
   d. Press ⊙ until ZoomStat is highlighted; press (ENTER) to display a graph with the *x* and *y* ranges set to fill the screen with data.
   e. Press (TRACE) to determine the coordinates of a point on the graph using the cursor keys.

⇒ Answer Question 1 on the *Data Collection and Analysis* sheet.

3. Plot an exponential model of $y = Te^{-kx}$ to the data. Use these new variable names to match what the calculator can use. You can determine the value for T (the initial temperature difference T_0) by tracing the graph to the point at $x = 0$. Record this parameter as T in the Data Table.

4. Since you have values for the parameter *T* of the model, you can try plotting the model using a guess for the *k* parameter. First, enter the model equation for graphing.

   a. Press ⬭Y=.
   b. Press ⬭CLEAR to remove any existing equation.
   c. Enter the exponential model using the value for the parameter *T*. For example, if the equation is $y = 25e^{-kx}$, enter 25*e^(–K*x) on the Y1 line.
   d. Press ⬭◄ until the icon to the left of Y1 is blinking. Press ⬭ENTER until a bold diagonal line is shown which will display your model with a thick line.
   e. Press ⬭2nd [QUIT] to return to the home screen.

5. Set a value for the parameter *k*, and then look at the resulting graph. To obtain a good fit, you will need to adjust the value of *k*. Use the steps below to store different values to the parameter *k*, starting with *k* = 0.01.

   a. Enter a value for the parameter *k*. Press ⬭STO► K ⬭ENTER to store the value in the variable K.
   b. Press ⬭GRAPH to see the data with the model graph superimposed. The model line is bold.
   c. Press ⬭2nd [QUIT] to return to the home screen.

   Experiment until you find one that provides a good fit for the data. Record the *k* value that works best in the Data Table.

⇒ Using this *k* value and the value of *T* you determined earlier, complete the model equation and record it in Question 2 on the *Data Collection and Analysis* sheet.

6. You can also use the calculator to fit an exponential to the data. The calculator uses the exponential form of $y = ab^x$. This is slightly different from the form you used in the modeling of the previous step, but it is still an exponential.

   a. Press ⬭STAT and use the cursor keys to highlight CALC.
   b. Press the number adjacent to ExpReg to copy the command to the home screen.
   c. Press ⬭2nd [L1] ⬭, ⬭2nd [L2] ⬭, to enter the lists containing the data.
   d. Press ⬭VARS and use the cursor keys to highlight Y-VARS.
   e. Select Function by pressing ⬭ENTER.
   f. Press ⬭ENTER to copy Y1 to the expression.
      On the home screen, you will now see the entry ExpReg L1, L2, Y1. This command will perform an exponential regression with L1 as the *x* values and L2 as the *y* values. The resulting regression equation will be stored in equation variable Y1. Press ⬭ENTER to perform the regression. Copy the parameters a and b that appear on the calculator screen to the Data Table.
   g. Press ⬭GRAPH to see your graph.

⇒ Answer Questions 3–9 on the *Data Collection and Analysis* sheet.

EXTENSION

Newton's law of cooling states that the rate of change of the temperature of a liquid is directly proportional to the difference between its temperature and the temperature of the surroundings. The differential equation is

$$\frac{dT}{dt} = -k(T - T_{room})$$

where T_{room} represents the surrounding temperature. Solve the differential equation (showing the steps) given above to show that $T = (T_0 - T_{room})e^{-kt} + T_{room}$. In this equation, T_0 represents the initial temperature.

DATA COLLECTION AND ANALYSIS

Name   _____

Date   _____

DATA TABLE

Minimum Temperature (°C)		

$y = Te^{-kx}$	T	
	k	
$y = ab^x$	a	
	b	

QUESTIONS

1. Is the graph consistent with the model of a decreasing exponential? In what way?

2. Record the model equation from Step 5 of the Analysis section here.

3. How does the regression fit compare to the model? How can both models fit the data well when the fit parameters are different?

4. Why are the values for *a* and *T* similar, while *b* and *k* are quite different? Use the model equations to find the relationship between *k* and *b* so you can compare the value to that of the calculator.

5. When $t = 0$, what is the value of e^{-kt}?

6. When t is very large, what is the value of the temperature difference? What is the temperature of the sensor at this time?

7. What could you do to the experimental apparatus to decrease the value of k in another run? What quantity does k measure?

8. Use either the model equation or the regression equation to predict the time it takes the sensor to reach a temperature 1ºC above room temperature.

9. If the starting temperature difference is cut in half, does it take half as long to get to 1ºC above room temperature? Explain.