EXPLORATIONS

Chapter 4

Integration

In this chapter, you will explore indefinite and definite integrals.

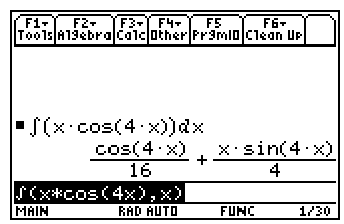
Calculus courses describe many techniques of integration, such as integration by parts. The TI-89 can be used to verify the solutions to most problems.

Example 1: An indefinite integral

Integrate $\int x \cdot \cos(4x)dx$.

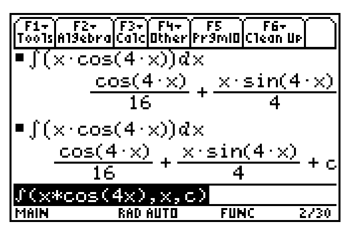
Solution

Use the integrate command (∫) on the Home screen.



1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.
2. Enter the integral. The multiplication symbol after the first *x* is important; otherwise, the expression will be interpreted as an undefined function **xcos**.

   [2nd] [∫] **X** [×] [2nd] [COS] **4 X** [)] [,] **X** [)] [ENTER]



3. Of course, the complete solution is a family of curves, generally indicated by +*c*. In addition, the constant will be useful for further work, such as substituting an initial condition. Therefore, obtain a solution with the constant *c*.

   [▶] [◀] [,] **C** [ENTER]

   Note that choices such as *c1* and *c2* cannot be used as they are reserved for the columns of the Data Matrix Editor.

Example 2: A definite integral

You can evaluate definite integrals with the TI-89, usually with an exact or approximate solution.

Evaluate

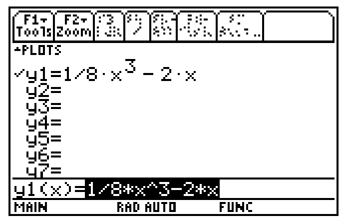
$$\int_{-2}^{3}\left(\frac{1}{8}x^3 - 2x\right)dx$$

Also, compute the area under $\frac{1}{8}x^3 - 2x$ on [-2,3].

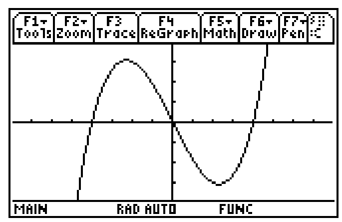
Solution

Use the integrate command (∫) on the Home screen and the Graph screen.

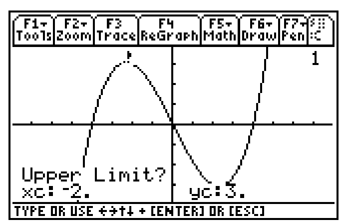
1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.

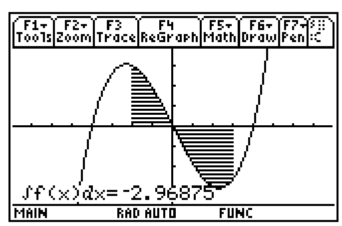


2. Press [◆] [Y=]. Clear any functions in the Y= Editor. With the cursor on $y1$, type the function and press [ENTER].
3. Press [F2] **Zoom** and select **4:ZoomDec** to graph the function.



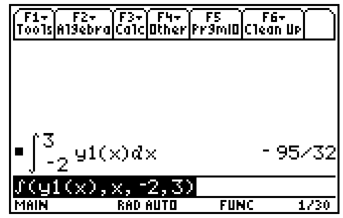
4. To evaluate the definite integral, press [F5] **Math** and select **7:∫ f(x)** d**x.** Type **⁻2** as the lower limit and press [ENTER]. Type **3** as the upper limit and press [ENTER].



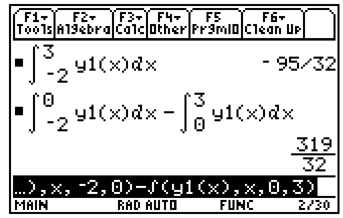


5. Press [HOME] to return to the Home screen and repeat the example.

   [2nd] [∫] **Y1** [(] **X** [)] [,] **X** [,] [(-)] **2** [,] **3** [)] [ENTER]



6. Since this function has a region below the x-axis on the interval [-2,3], the result for the area under the curve is not the same as the result for the definite integral computed above. There are several methods that can be used to compute the area. Since the curve is above the x-axis for [-2,0} and below the x-axis for [0,3], you can compute

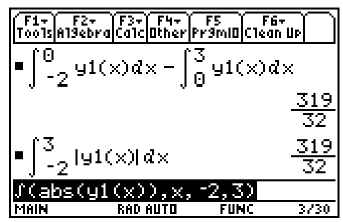


$$\int_{-2}^{0} y1(x)dx - \int_{0}^{3} y1(x)dx$$

   [▶] [◀] [←] **0** [▶] [−] [2nd] [∫] **Y1** [(] **X** [)] [,] **X** [,] **0** [,] **3** [)] [ENTER]

7. Alternately, you can compute this area by integrating the absolute value of $y1(x)$ on [-2,3] using **abs(**.

   [2nd] [∫] [CATALOG] **abs(Y1** [(] **X** [)] [)] [,] **X** [,] [(-)] **2** [,] **3** [)] [ENTER]



Example 3: An integral formula

Since the TI-89 has a symbolic algebra system, integrals with undeclared coefficients can be computed. The results resemble those found in tables of integrals. They are beneficial for generating formulas and pattern recognition.

$$\text{Integrate } \int \frac{1}{a^2 + (bx)^2} dx .$$

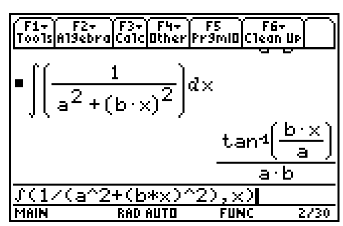
Solution

Use the integrate command (∫) on the Home screen.

1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.

2. Enter the function. Recall that it is important to type **b*x** not **bx**. You can also use undeclared variables in the limits.

   [2nd] [∫] **1** [÷] [(] **A** [^] **2** [+] [(] **B** [×] **X** [)] [^] **2** [)] [,] **X** [)] [ENTER]



Example 4: Symbolic limits in a definite integral

$$\text{Evaluate} \int_a^b \frac{1}{k+t}\,dt$$

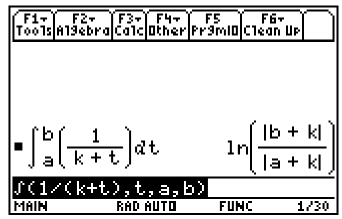
Solution

Use the integrate command (∫) on the Home screen.

1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.

2. Enter the definite integral.

   [2nd] [∫] **1** [÷] [(] **K** [+] **T** [)] [,] **T** [,] **A** [,] **B** [)] [ENTER]



Exercises

Integrate each example.

1. $\int \frac{x}{1+4x^2}dx$
2. $\int a \cdot b^{k \cdot t} dt$
3. $\int_0^{\frac{\pi}{3}} \sin(x)\cos(x)dx$
4. $\int_p^q (m \cdot x + b)^n dx$