

Transformations of Functions

Teacher Notes and Answers

7 8 9 10 11 12



Introduction

Functions can be reflected, dilated or translated, collectively these are called transformations. The purpose of this activity is to focus on the transformations. Data has been collected using a variety of sensors to measure the following:

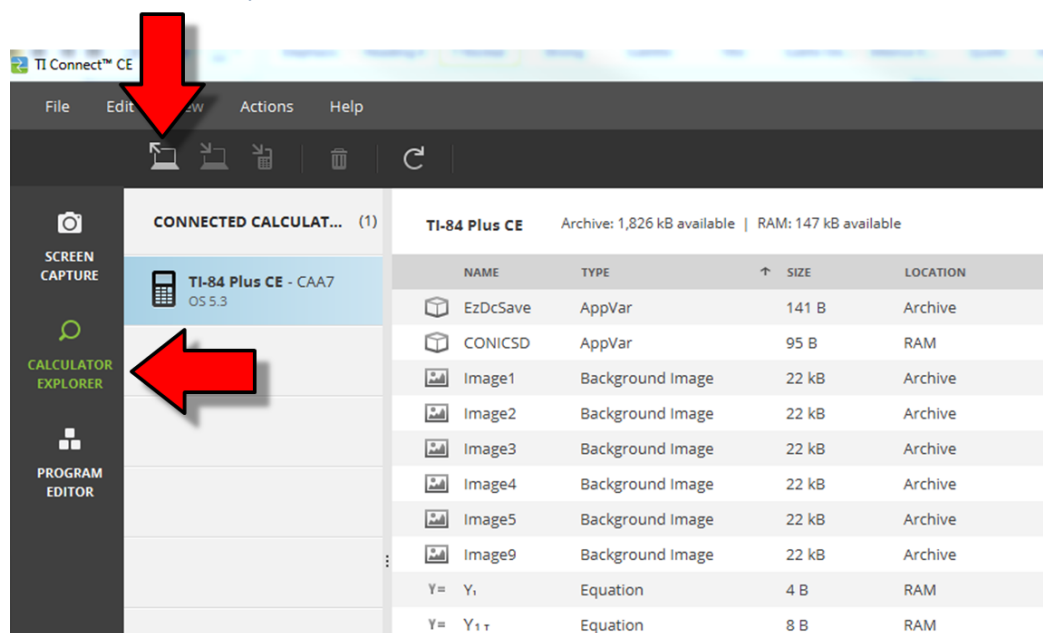
- Motion of a bouncing ball
- Motion of a pendulum
- Discharge of a capacitor

For each of these sets of data, an appropriate function has been defined in Y_1 . Transformations to this function can be applied using Y_2 .

Teacher Notes:

The group file can be transferred to student calculators using the FREE TI-Connect CE software.

- Download the file from the Texas Instruments Australia website.
- Connect your TI-84PlusCE calculator to your computer and launch the TI-Connect CE software.
- Click on the Calculator Explorer and the Computer to Calculator option
- Locate the file on your computer and follow the prompts.



Note that the aim of this activity is to help students connect the basic transformations to a range of functions. The pendulum data is a trigonometric function, however students do not need to know to calculate the period of a trigonometric function as the focus is on the general effect of dilating the function parallel to the x axis.

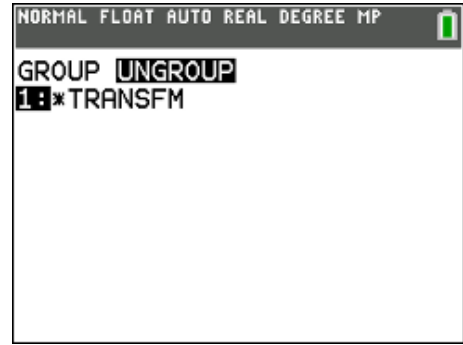
Similarly with the exponential data, a precise equation can be determined using the ratio between consecutive terms in the data, however students need to consider appropriate reflections and dilations in order to establish the best equation to model the data.

The Group file: TRANSFM contains the data sets and three programs, one for each data set to help set up the calculator.

To open the Group file from the home screen press:

2^{nd} [mem] – Select Option 8 [Group] then Ungroup

The group file contains lists: $L_1 \dots L_6$ and some program files. When prompted at the “Duplicate Name” menu, select “Overwrite All”



Calculator Tip!



The investigation jumps in and out of the graph screen. To speed up the graphing process the XRES (x resolution) has been changed to 3, this means graphs may appear slightly distorted as fewer points are used to graph each function. When you want to see the graph in more detail change the XRES in the window settings back to the default value of 1.

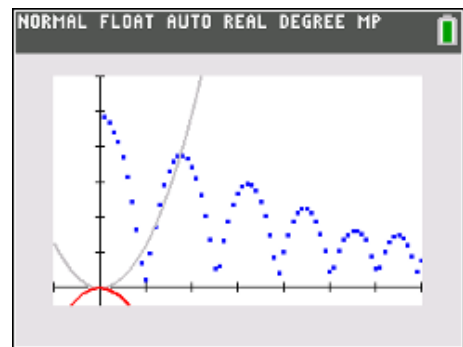
Teacher Notes:

Other activities available for free from the Texas Instruments Australia website are available for capturing data to ensure students have their own unique data sets. This activity use the CBR2 to collect the data.

Bouncing Ball

Run the program “Prob1”. This will select the data for the ball bounce problem, set up a scatter plot and enter the basic function in Y_1 .

In Y_2 enter the equation: $-Y_1$.



Question 1

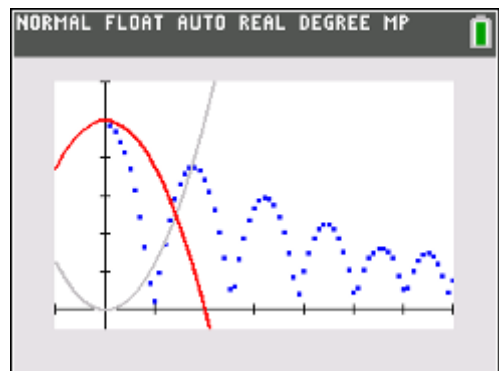
How does this first transformation affect the graph?

Answer: The graph is a reflection of the original function in the x axis.

The next step is to provide explore another transformation of the function. Edit the function defined by Y_2 as $-Y_1 + k$ by trying different values of k with an attempt to model the first ‘drop’ of the bouncing ball.

Note:

It is not essential at this stage to get the ideal value of k , something relatively close is sufficient as it can be edited again later.



Question 2

How does this transformation affect the graph?

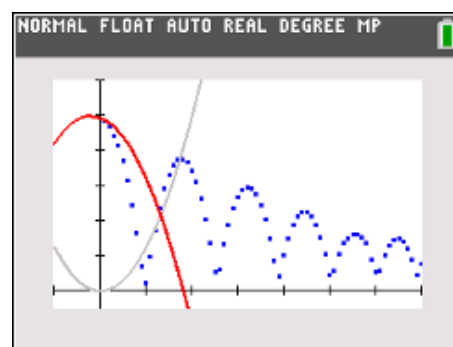
Answer: It is a translation parallel to the y axis.

The next step is to provide explore another transformation of the function. Edit the function again, keeping all previous transformation:

$-Y_1(x - h) + k$ by trying different values of h .

Note:

Once again, it is not essential at this stage to get the ideal value of h , something relatively close is sufficient as it can be edited again later.

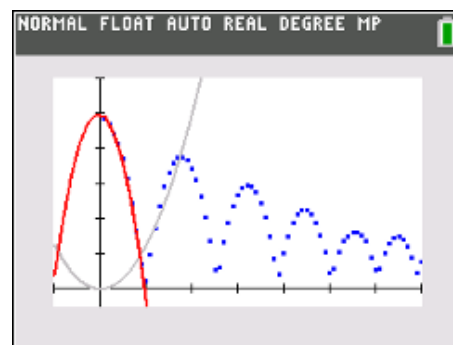
**Question 3**

How does this transformation affect the graph?

Answer: It is a translation parallel to the x axis.

The last transformation, keeping all previous transformation is to edit the value of a in: $-aY_1(x - h) + k$

Once you have explored possible values of a , edit some of the previous transformations as necessary and try and provide the best match possible for the first drop of the bouncing ball.

**Question 4**

How does this transformation affect the graph?

Answer: It is a dilation. Students may say it makes the graph 'wider' or 'narrower', these terms however should be used with caution as the graph is 'infinitely' wide, our view through the window settings however corresponds to the description of 'wider' or 'narrower'.

Calculator Tip!



To get a closer view of the first ball bounce, use the Zoom option and use Zoom Box. Draw a box around the data and graph that you want to fill the screen. Edit the equation to get the best fit possible. When you are happy with your equation, use the Zoom – Memory option and select “Previous” to return to the original window settings.

Question 5

Write down the overall equation for your graph, using function notation, that models the first drop of the bouncing ball?

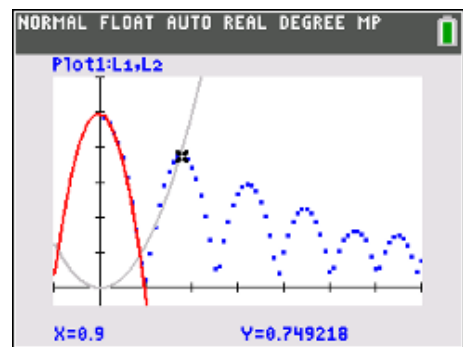
Answers will vary, the function: $f_2(x) = -4f_1(x) + 0.98$ provides a suitable approximation for the first ball bounce.

The remaining ball bounces can be modelled using the same approach as above.

Use Y_3 in terms of Y_1 to model the second ball bounce and so on with the remaining ball bounces.

The TRACE option can be used to determine specific Time and Position coordinates to assist in the transformations. Remember that the data is real so specific points may not represent the ideal location for the turning point on any given ball bounce / quadratic function.

The calculator screen shown opposite shows the approximate location of the peak height for the second ball bounce.



Calculator Tip!



The TRACE option automatically stores the current X and Y coordinates in variables X and Y. These values can be recalled using $\boxed{2nd} [rc1]$ and selecting the corresponding variable. This is a very efficient way to translate the function!

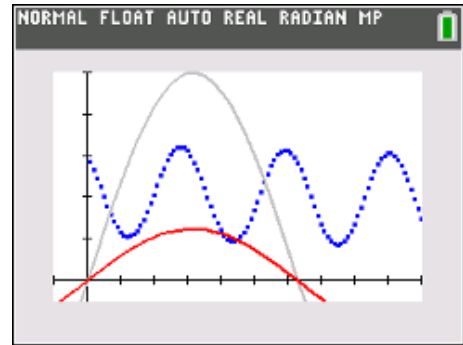
Question 6

Determine equations for the remaining ball bounces.

Answer: Answers will vary, however the value of ‘a’ in each will be approximately -4.9 (half of gravity) and the translations can be approximated by looking at the peak of each ball bounce using the trace function.

Pendulum

Run the program “Prob2”. This will select the data for the pendulum swing problem, set up a scatter plot and enter the basic function in Y_1 . In Y_2 enter the equation: $0.25Y_1$.



Question 7

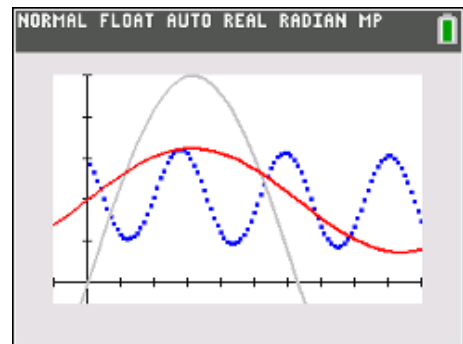
How does this first transformation affect the graph?

Answer: Dilation, changes the amplitude of the function. (Dilation parallel to y axis)

The next transformation is of the form:

$$aY_1 + k$$

Explore the values of k that provide a reasonable transformation. Once again, this is approximate, all these transformations can be edited as the modelling function gets closer to the actual data.

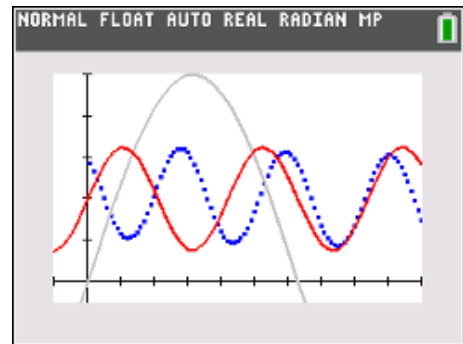


Once you are happy with the previous transformation explore and edit the following transformation:

$$aY_1(n(x - h)) + k$$

Start with a value of $n = 3$.

At this stage it is appropriate to edit the previous two transformations.

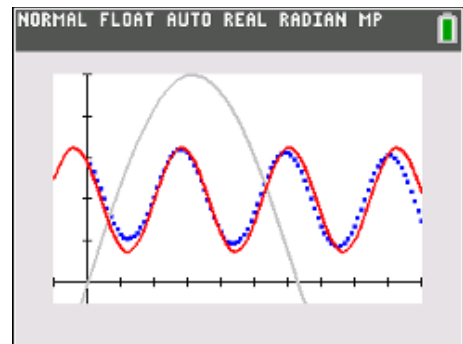


The final transformation for this function is:

$$aY_1(n(x - h)) + k$$

Make sure all parentheses are placed appropriately. There are numerous value of h that will work for modelling the pendulum. Identify one and work on the specific value.

As your function gets closer and closer to the data, edit each parameter accordingly.



Question 8

Record the function that best models the pendulum data.

Answer: One function that models the data reasonably well: $f_2(x) = 0.23f_1(4(x-1)) + 0.41$

Note that infinitely many other solutions exist as the function may be translated parallel to the x axis in infinitely many ways. It is also important to note that this is genuine data, so precise modelling is not possible due to the genuinely practical nature of the task.

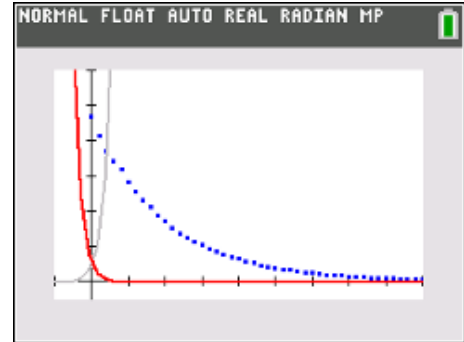
Capacitor

Run the Prob3 program. The electrical discharge of a capacitor has been recorded. A knowledge of electronics is not required for this task, however it is nice to know just how precisely electronics can align to mathematical equations!

An exponential function has been defined in: Y_1 .

Use Y_2 to explore transformations of the form:

$$f_2(x) = af_1(bx)$$



Question 9

Record the function that best models the discharge of a capacitor.

Answer: $f_2(x) = 9.5f_1(-0.1x)$ $Y_2 = 9.5Y_1(-0.1X)$