Transforming Relationships

ID: 11525

Time Required
40 minutes

Activity Overview

In this activity, students will assess the strength of a linear relationship using a residual plot. They will also calculate the correlation coefficient and coefficient of determination to assess the data set. Students will then learn to transform one or two variables in the relationship to create a linear relationship.

Topic: Two-Variable Statistics

- *Residual plots*
- *Transforming relationships*
- *Inverse functions*
- *Linear, logarithmic, exponential, and power function graphs*

Teacher Preparation and Notes

- *The activity is designed as a class activity. The teacher document has questions that should be asked to lead the discussion. More background information may be needed for some classes depending on their knowledge of inverse functions and logarithms.*
- *The extension can be used or omitted, depending on the ability of the class.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11525" in the keyword search box.***

Associated Materials

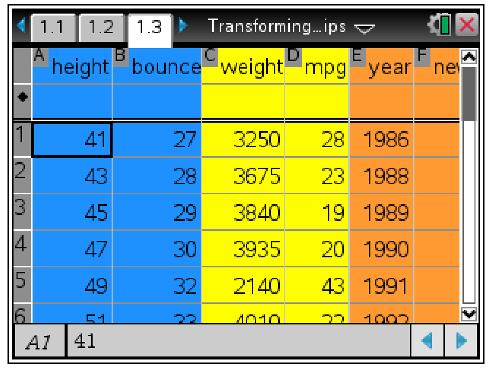
- *Transforming_Relationships_Student.doc*
- *Transforming_Relationships.tns*
- *Transforming_Relationships_Soln.tns*

Suggested Related Activities

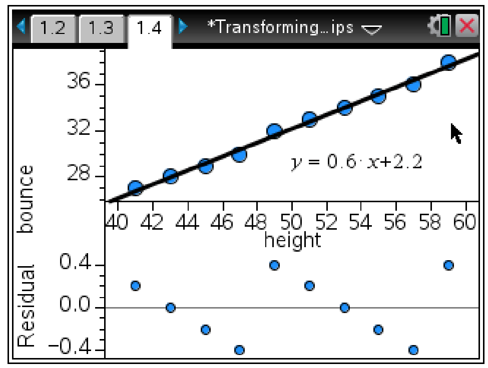
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Square It Up! (TI-Nspire technology) — 11408*
- *What is Linear Regression? (TI-84 Plus family) — 6194*
- *What's My Model? (TI-Nspire technology) — 8518*

Problem 1 – Analyzing Residual Plots



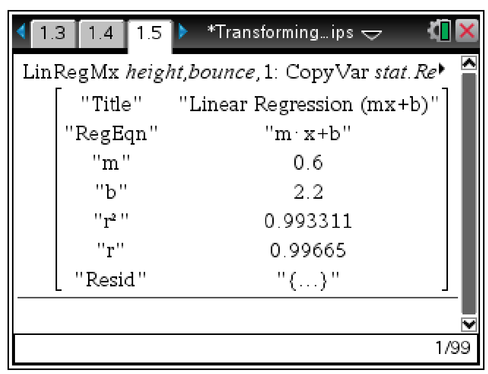
Students are given a spreadsheet with four data sets. Each data set is graphed on a separate scatter plot. Students are asked to create a linear regression model and assess the fit graphically and numerically.



The first two graphs are obviously linear. The third one is not a good fit. The residuals do not show a curved pattern, which means that the linear model is most appropriate. While the linear model is not good, there is still no other model that works better. Discuss this point with students. In addition, the *r*-value in graph four is good, but the residual plot shows a curved pattern.

TI-Nspire Navigator Opportunity: *Class Capture*

See Note 1 at the end of this lesson.

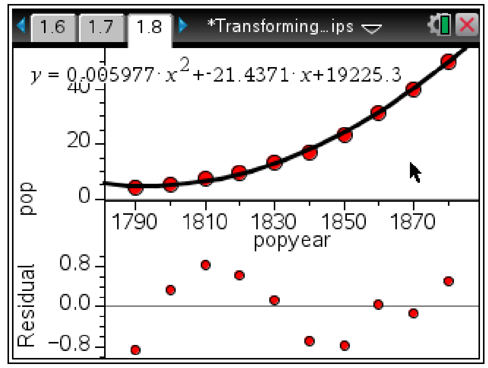


Point to emphasize: A low *r*-value means that there is a poor fit in a linear model. Together, the residual plot and the *r*-value help determine if a non-linear model is more appropriate.

More discussion can take place to determine what each regression line specifically means. For example, the regression line with bounce vs. height tells us that they are related linearly. Specifically, the regression equation tells us that for each additional unit of height, the bounce increases by 0.6 units.

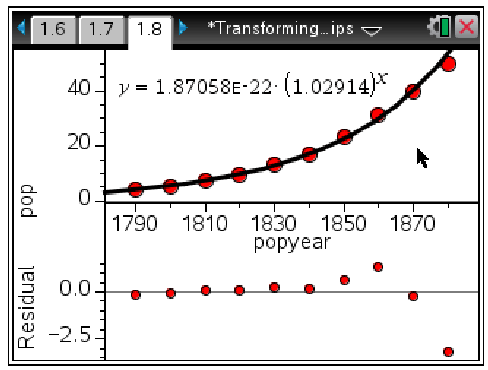
Problem 2 – Transforming Data

Allow students the opportunity to play with the models, and then explain which model best represents the data. Some may think that a quadratic model would work, while others might think that an exponential model would work. However, they should see that a quadratic model may not be ideal because the residual plot produces a pattern.

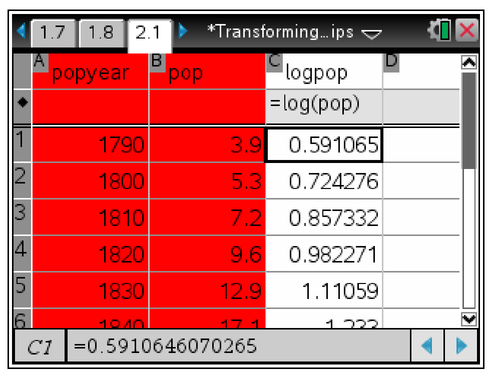


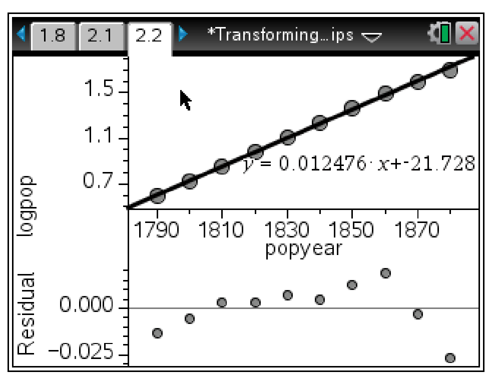
Discussion can then focus on population growth and exponential modeling.

After the exponential model is agreed upon, guide students in a discussion on how to make an exponential function into a linear function. You may need to guide the discussion towards a logarithmic function.



Students are to calculate the logarithm of the population on the spreadsheet on page 2.1. Then, they are to graph logpop vs. popyear on a new *Data & Statistics* page and carry out the assessment of the linear model.





Good questions to ask:

- *What does an inverse function do?*
- *What is the inverse of an exponential function?*
- *Will the base of the logarithm make a difference?*

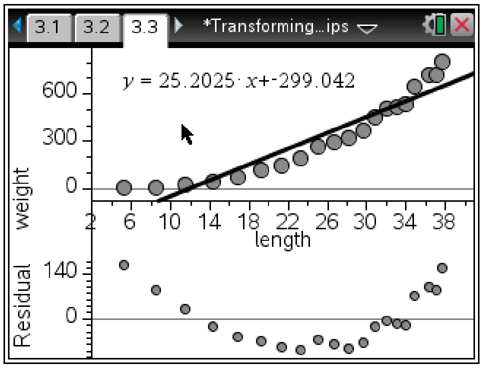
(This last question will not be considered in this activity. However, it would be an excellent extension or quick class investigation because the TI-Nspire technology allows students to readily choose the base.)

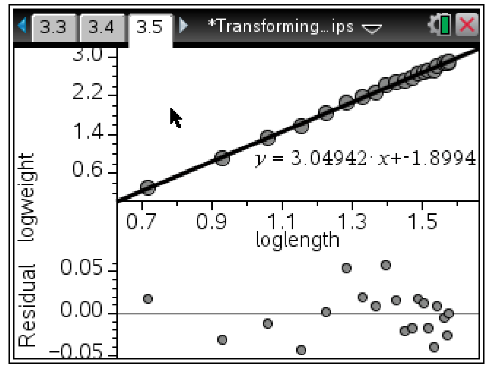
More discussion can take place regarding transformations and power functions. Many textbooks contain more thorough discussion and background information.

If the linear model fits the transformed graph well, then the regression model picked for the original data also fits well.

Extension – An Additional Transformation

The data given will be best modeled by a power function (i.e., exponential). This means that both variables will need to be transformed using a logarithmic transformation in order to create a linear model.





Solutions to Student Worksheet

	Independent variable	Dependent variable
Bounce and Height	Height	Bounce
MPG and Weight	Weight	MPG
Tons of paper and Year	Year	Tons of paper
Population and Year	Year	Population

1. The linear regression line fits the data because there is no obvious curved pattern. Some points are above the line and others are below.
2. r = 0.99665 and r^2 = 0.993311. These values tell you that the data closely models a linear relationship since they are close to 1.
3. The linear model fit well because the residual plot, r, and r^2 all support this.
4. For every additional unit increase in height, the bounce increases by 0.6 units.

	Graphically	Numerically
mpg vs. weight	Even distribution above and below. No obvious pattern	r = –0.959483 r^2 = 0.920607
newspaper vs. year	Even distribution above and below. No obvious pattern	r = 0.576045 r^2 = 0.331828
pop vs. popyear	Obvious curved pattern	r = 0.95641 r^2 = 0.914742

5. Population vs. Population year
6. Exponential
7. Exponential because it goes through all but one data point.

8. Yes, it appears to be linear.
9. Graphical: While it looks like there may be a pattern to the residual plot, looking at the values of the residuals, they are very close to zero.

   Numerical: r = 0.999402, r^2 = 0.998805.

Extension Solutions

1. The residuals show a very definite curved pattern. r = 0.946104 and r^2 = 0.895112
2. Power function
3. The residuals show a very definite curved pattern. This time the curve is upside down. R = 0.840227 and r^2 = 0.705982
4. The residuals show no pattern and have very small magnitudes. r = 0.999261 and r^2 = 0.998523
5. A power function is a good model for the data.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Class Capture*

This would be a good place to do a Class Capture to compare student results. Examine the movable lines students used to find the regression equation of each set of data.