## Limit of $\operatorname{Sin}(x) / x$

Time required
ID: 11067
15 minutes

## Activity Overview

In this activity, students will graph $f(x)=\frac{\sin x}{x}$ in order to visually determine the limit as $x \rightarrow 0$.
They will confirm the answer numerically by tracing left and right limit points to capture values in a spreadsheet.

## Topic: Limits

- Trigonometric functions
- Left-hand and right-hand limits
- Graphical limits


## Teacher Preparation and Notes

- With the accompanying worksheet, this TI-89 activity has step by step instructions so students can explore $\sin (x) / x$ graphically, numerically and algebraically.
- A connection to the formal limit notation is considered to connect formal mathematical notation to visual representation using CAS.
- For set-up before the activity begins, have students complete the following:
o Press MODE and set Graph to FUNCTION and Angle to RADIAN. Press ENTER to save the changes, if any.
o Turn the functions off or clear them. To turn clear the functions, on the $Y=$ screen press F1 and select Clear Functions. To turn off the functions, uncheck them by highlighting the function and pressing [F4].
- The first two parts of this activity can be done with the TI-84 Plus family.
- To download the student worksheet, go to education.ti.com/exchange and enter "11067" in the quick search box.


## Associated Materials

- CalcWeek03_LimitSinX_worksheet_TI-89.doc


## Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- Limits — 8997
- Sequences in Calculus -8665
- Black Bear Cubs From Birth to One Year Old - 8317
- Exponential Differentiation -8980


## Part 1 - Graphical Limit

At the beginning the activity, the student is introduced/reminded of the concept of indeterminate. Have them consider what $\sin (0)$ equals to understand that $y(0)=\frac{\sin (0)}{0}=\frac{0}{0}$.

Students are to graph the function by typing $\boldsymbol{\operatorname { s i n }}(\mathrm{x}) / \mathrm{x}$ next to y 1 in the $\mathrm{Y}=$ screen. Viewing the graph, they need to quickly approximate $y 1(x)$ as $x$ approaches 0 . The tick marks occur every 0.5 units on the $y$-axis.

By turning off the axes, students can see the discontinuity of the function.

Trace is then used to see the values that correspond to the graph. Values can be inputted while Trace is active by simply typing a number like 0.1.

Tip: The last xc and yc values are automatically stored in case you want to recall the value of the $x$ and $y$ coordinates for a calculation on the HOME screen. To see this press HOME, type $\square$ alphan ENTER.

## Part 2 - Numerical Limit

Students are now led to set up the table and investigate the values.

In order to emphasize that for a limit to exist it must be approaching the same value from both sides, students will move the curser to examine both sides of zero.


## Part 3 - Algebraic Limit

Students are to go to the home screen. If work was previous done, they may need to clear the screen. To do this, press 2nd F1 and select New Prob. Once this is on the command line, press ENTER to execute.

They are to find the left-hand, right-hand and regular limits.

## Practice Problems

Students are given extra practice problems to complete using what they have learned in this activity. They can do the problems for homework or if there is extra time in class.

To use the split-screen, press MODE and set 'Split Screen' to LEFT-RIGHT, 'Split 1 App' to Graph and 'Split 2 App' to Table. Press 2nd APPS to
 move between the graph and table.

1. $\lim _{x \rightarrow 1} \frac{x-1}{x^{3}-1}=\frac{1}{3}$
2. $\lim _{x \rightarrow 0} \frac{1-\cos (x)}{x^{2}}=\frac{1}{2}$
3. $\lim _{x \rightarrow 0}(1+x)^{\frac{1}{x}} \approx 2.72$
