Optimal U-Turn

STEM Lesson for TI-Nspire™ Technology

Objective: Collect data, analyze the data using graphs, and use the results to determine the best driver.

About the Lesson: On a drag strip the shortest path is the fastest path: a straight line from start to finish. On the U-turn track however, the fastest path is not usually the shortest path. A driver should 'round' the corners just like a baseball player 'rounds' the bases. In both cases it is a longer path. The type of surface you race on is also a factor in the fastest path. Keep these facts in mind and choose a drive path that doesn't make you skid:

1. Skidding is slower than smooth movement
2. You have to accelerate more when you skid and acceleration requires a lot of energy.

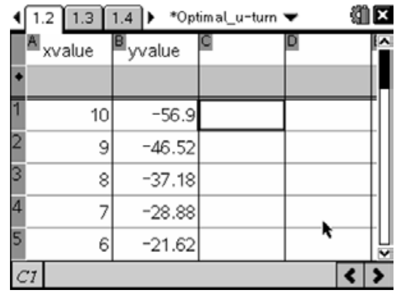
Materials: RC Car
Masking tape (if not on a tiled floor)
U-Turn track: 50 ft. to the turnaround point and a 10 ft. lane
Cones or plastic cups
Scaled graph paper
Student Worksheets

Procedure:

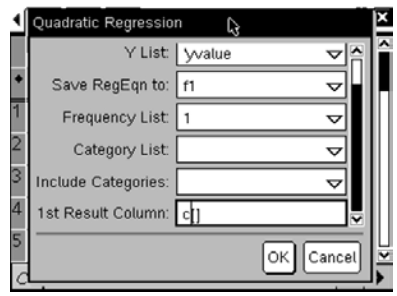
1. Using masking tape or a tiled floor, create a Cartesian plane that is [-10,10] x [-50,10].
2. Use the y-axis as the center of the track and the turn around point location at (0,0), and construct a u-turn track 50-foot long with 10-foot lanes.
3. Choose one driver and several recorders. The recorders will need a sheet of scaled graph paper.
4. The driver will drive a single lap while the recorders watch the car and plot estimated points where the car crosses the Cartesian plane. Each recorder should record 5-7 points.
5. On the scaled grid provided in the student worksheet, combine all of the points for one driver. Sketch a parabola through the points.

Analysis:

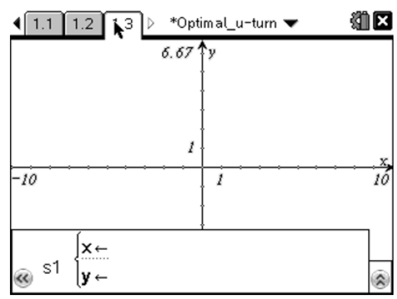
1. On your handheld, go to My Documents and open the file named *Optimal_U-turn.tns.*



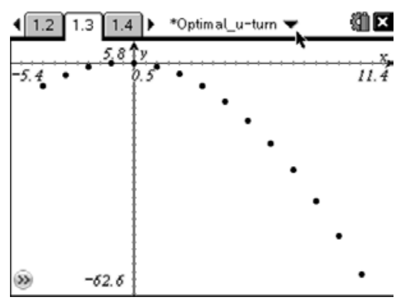
2. Use ctrl ▶ to move to page **1.2**. Enter your *x* and *y* values into column A and B.



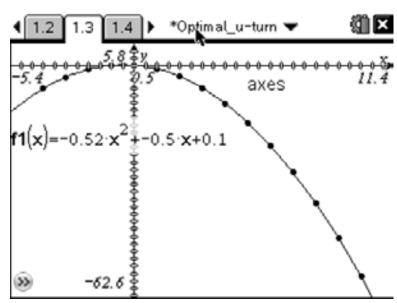
3. Do a quadratic regression analysis on your data. Press menu then choose **Statistics > Stat Calculations > Quadratic Regression**. The regression template will pop up. Remember to press tab to switch boxes. Click the arrow at the right of each box to access the drop down menus. Choose *xvalue* for the X List and *yvalue* for the Y list. Tab through the other options until you get to 1st Result Column. Click beside the letter and change it to 'c' if it isn't already. Click OK.



4. Move to page **1.3**. Press menu then choose **Graph Type > Scatter Plot**. Notice the function bar at the bottom of the screen changes from f1 to s1.



5. Press var and choose x. Press ▼ then var and choose y. Press enter. Now adjust your window to view the data by pressing menu then choosing **Window/Zoom > Zoom – Data**.



6. Now graph the regression line. Press menu then choose **Graph Type > Function**. Press ▲ to see your function in f1 then press enter. You should see your quadratic function on top of your data points.

7. What is the equation of your function?

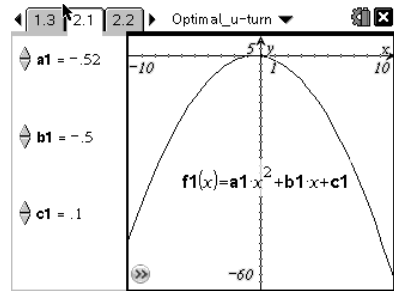
8. Move the cursor over the graph of the function until the double arrow pops up and press ctrl then (in the center of the touchpad). Now stretch the graph away from the data points.

   What happens to the equation?

9. When a quadratic function is in the form of $y = Ax^2 + Bx + C$, it is in standard form Stretch the graph 4 more times and fill in the chart below with the equation and A, B, and C.

EQUATION	A	B	C

Move to page **2.1**. This graph shows a sample drive path for the 50-foot track.

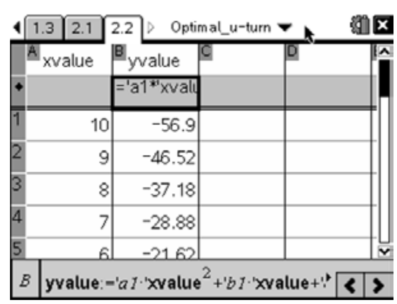


10. Click on the △ and ▽ on the left side of the screen next to the variable *a1* to change the value of *a1*. What happens to the graph?

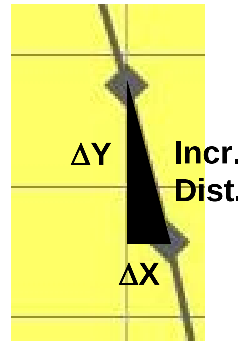
11. Click on the △ and ▽ on the left side of the screen next to the variable *b1* to change the value of *b1*. What happens to the graph?

12. Click on the △ and ▽ on the left side of the screen next to the variable *c1* to change the value of *c1*. What happens to the graph?

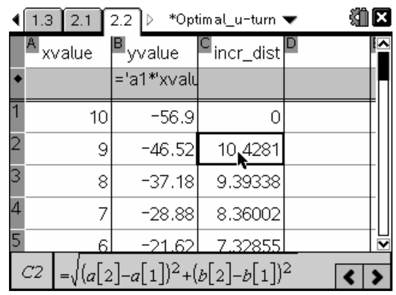
Move to page **2.2**. This table shows the x and y values from your graph on page **2.1**.



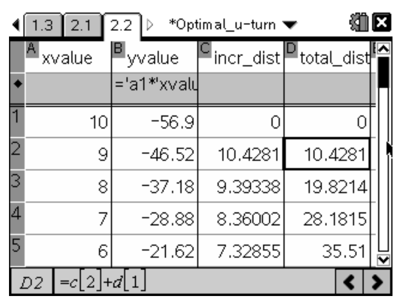
To calculate the fastest drive path, you need to break down the track into increments. Since the track was setup as a Cartesian plane, remember the x-value is the horizontal position on the track; and the y-value is the vertical position on the track. Using the center of the track as x = 0, the domain is [-10,10] for 10-foot lanes; and the start and finish lines are at y = -50 and y = 0 for the turn around point for a 50-foot long track. Don't forget the car can go up to 10 ft past y = 0 due to the lane width.



13. Move to the top of column C. Type **incr_dist** to name the column. Next click the ▼ button to move down row 1. Use the Pythagorean Theorem to estimate the incremental distance between points. This means Incr. Dist. = √(Δx^(2)+Δy^(2)). Enter 0 in the first row because the car has not traveled at this point. Then move to row 2 and press (=)(ctrl)(x^2) then type (a[2]-a[1]). Press (x^2) for the exponent then continue typing +(b[2]-b[1]) and press (x^2) and (enter). Then press (menu) and click **Data >> Fill Down**. Press ▼ until you get to the bottom of the list and press (enter).



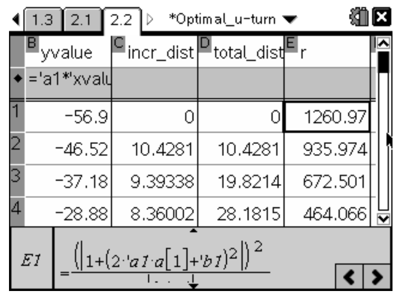
14. Now move to the top of column D and name it **total_dist**. This will be the total distance that the car has traveled since the starting line. Next click the ▼ button to move down row 1. Enter 0 in the first row because the car has not traveled at this point. Then move to row 2 and press (=) then type c[2]+d[1]. Then press (menu) and click **Data >> Fill Down**. Press ▼ until you get to the bottom of the list and press (enter).



Now we need to calculate the radius of curvature using our equation for the drive path:

$$rad(r_x) = \frac{\left(\left|1+(2ax+b)^2\right|\right)^{3/2}}{|2a|}$$

15. Move to column E and title it **r**. Enter the equation in row 1 like before using the variables *a1* for a and *b1* for b. Use a[1] for x. Since a[1] is a column and *a1* is a variable, the handheld will ask you which one you want to use. Choose **Cell Reference** when using the cells of the spreadsheet and **Variable** when using the *a* from the graph. Make sure to press the multiplication key in between variables. Fill down.



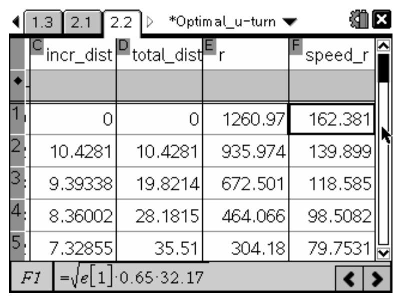
	B yvalue	C incr_dist	D total_dist	E r
◆	='a1*'xvalu			
1	-56.9	0	0	1260.97
2	-46.52	10.4281	10.4281	935.974
3	-37.18	9.39338	19.8214	672.501
4	-28.88	8.36002	28.1815	464.066

E1 $=\left(\left|1+(2\cdot 'a1\cdot a[1]+'b1)^2\right|\right)^2$

Next we need to find the speed limit base on the radius using the equation

$V = \sqrt{rC_Fg}$ where $C_F = 0.65$ and $g = 32.17$ ft/s^2.

16. Move to column F and title it **speed_r**. Type in the equation using e[1] for r and substituting the constants. Fill down.



	C incr_dist	D total_dist	E r	F speed_r
◆				
1	0	0	1260.97	162.381
2	10.4281	10.4281	935.974	139.899
3	9.39338	19.8214	672.501	118.585
4	8.36002	28.1815	464.066	98.5082
5	7.32855	35.51	304.18	79.7531

F1 $=\sqrt{e[1]\cdot 0.65\cdot 32.17}$

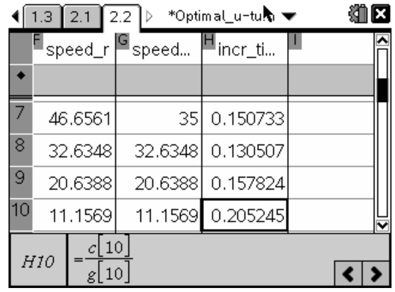
To calculate the total lap time, we need to consider the time it took to get to top speed, the distance to top speed, and the fact that the car will only go 35 ft/s.

Time to top speed, t_a
= (35 – 0) ÷ (t_a – 0)
= 19.3 ft/s/s; t_a = 1.81 sec

Distance to top speed, d_{ACCEL}
= ½ (top speed x t_a)
= 0.5 x 35 x 1.81 = 31.7 ft

17. Title the next two columns **speed_limit** and **incr_time**.

    For the speed limit, enter 35 ft/s in the rows where Speed R is greater than 35, and enter Speed R in the rows where it is less than 35.

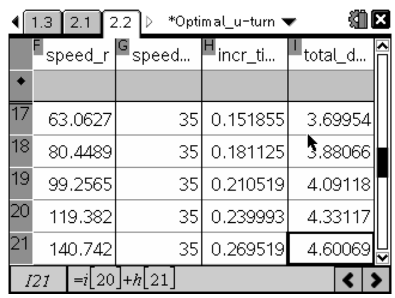


	F speed_r	G speed...	H incr_ti...	I
7	46.6561	35	0.150733	
8	32.6348	32.6348	0.130507	
9	20.6388	20.6388	0.157824	
10	11.1569	11.1569	0.205245	

H10 $=\frac{c[10]}{g[10]}$

    For the incremental time, enter 1.81 s in the first row where the total distance is greater than 31.7 ft. and use the formula incr. dist./speed limit (column c/ column g) to finish the column.

18. Finally move to column I and title it total_drive_time. Start with the 1.81 s in row 5. In row 6 enter the formula i[5]+h[6] and fill down.



	F speed_r	G speed...	H incr_ti...	I total_d...
17	63.0627	35	0.151855	3.69954
18	80.4489	35	0.181125	3.88066
19	99.2565	35	0.210519	4.09118
20	119.382	35	0.239993	4.33117
21	140.742	35	0.269519	4.60069

I21 =i[20]+h[21]

19. How long did it take the car to complete a lap?

20. Go back to page **2.1** and change the coefficients, then look at the Total Drive Time for the lap. Remember to adjust your 35 ft/s speed limit because your car has a maximum speed no matter what the calculations say. Repeat several times to fill in the table below.

A	B	C	EQUATION	TOTAL DRIVE TIME

21. Which drive path gives you the fastest (lowest) drive time?