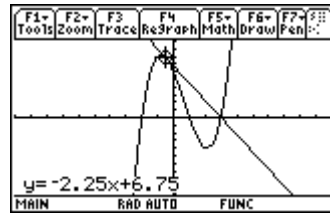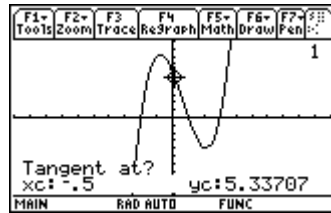# Cubics, Zeros and Tangents

Graph $y = (x + 2)(x - 1)(x - 3)$. Two of the roots of this function are $x = -2$ and $x = 1$. Graph the tangent line at the average of these roots. What is the root of this tangent line?

The root appears to be $x = 3$. This is the other root of the cubic function. Does this work for the other roots? Graph the tangent line at the average of $x = -2$ and $x = 3$. Then graph the tangent line at the average of $x = 1$ and $x = 3$.

===.

In each case the tangent line at the average of two of the roots appears to go through the other root of the cubic function.

Does this always work? Try several other third degree polynomials. Does the pattern continue for other examples? Make a conjecture and prove it. The proof is lengthy with paper and pencil. Prove this theorem with the computer algebra system found on a TI-89 or TI-92+ calculator. Should this still be considered an analytic proof?