Math Objectives

- Students will explore piecewise graphs and determine conditions for continuity and differentiability.

Activity Type

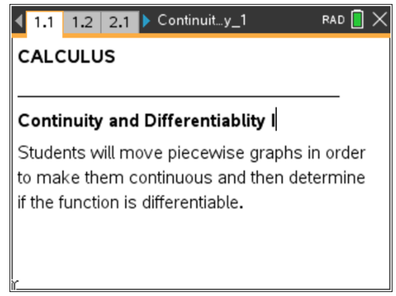
- Student Exploration

About the Lesson

- Students will complete a worksheet that discusses continuity and differentiability of each function in the file.

Directions

- For each of the following problems, increase or decrease the value of the slider by pressing the click button until there is a continuous function. Record the value of the slider variable and then differentiate the function (by hand) to determine if the function is differentiable. There may be more than one answer for a question. Students should attempt to solve the problem visually and then confirm answers algebraically.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Continuity_and_Differentiability_1_Student.pdf

Continuity_and_Differentiability_1_Student.doc

TI-Nspire document

Continuity_and_Differentiability_1.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

PART I:

For each question write the function with the values of the slider variable(s) that make the function continuous. There may be more than one answer. Confirm your solutions algebraically.

Problem 2.1:

Answer: $\mathbf{f}(x)=\begin{cases}-x, & x>-1\\ -x, & x<-1\end{cases}$

Problem 3.1:

Answer: $\mathbf{f}(x)=\begin{cases}-x, & x>1\\ -x^2, & x<1\end{cases}$

Problem 4.1:

Sample answers: $\mathbf{f}(x)=\begin{cases}x^2, & x>0\\ -2x^2, & x<0\end{cases}$ or $\mathbf{f}(x)=\begin{cases}(x-0)^2, & x>0\\ -2x^2, & x<0\end{cases}$

Problem 5.1:

Sample answers: $c-2=-4+d$

One possible solution: $\mathbf{f}(x)=\begin{cases}2x^2-2, & x>1\\ -4x^2+4, & x<1\end{cases}$

Another possible solution: $\mathbf{f}(x)=\begin{cases}x^2-2, & x>1\\ -4x^2+3, & x<1\end{cases}$

Problem 6.1:

Sample answers: $d=-c$

One possible solution: $\mathbf{f}(x)=\begin{cases}-2x^3+1, & x>-1\\ -x^3+2, & x<-1\end{cases}$

Another possible solution: $\mathbf{f}(x)=\begin{cases}x^3+1, & x>-1\\ -x^3-1, & x<-1\end{cases}$

Problem 7.1:

Answer: No value exists to create a continuous function.

Extension: Go back to each problem and determine if there is more than one solution that will result in a continuous function. If so, explain whether or not the additional solutions produce a differentiable or non-differentiable function.

Answer: Answers may vary.

PART II:

Calculate the derivative of each function using the values from part I and determine if the function is differentiable. Explain your conclusion.

Problem 2.1:

Answer: $\mathbf{f}'(x) = -1$

Problem 3.1:

Answer: Not differentiable because there is a cusp at $x = -1$.

Problem 4.1:

Answer: $\mathbf{f}'(x) = \begin{cases} 2x, & x > 0 \\ -4x, & x < 0 \end{cases}$

Problem 5.1:

Answer: The only value where **f**(x) is differentiable is when $c = -4$ and $d = -2$.

$$\mathbf{f}(x) = \begin{cases} -4x^2 - 2, & x > 1 \\ -4x^2 - 2, & x < 1 \end{cases}$$

$$\mathbf{f}'(x) = -8x$$

Problem 6.1:

Answer: The only value where **f**(x) is differentiable is when $c = -1$ and $d = 1$.

$$\mathbf{f}(x) = \begin{cases} -x^3 + 1, & x > -1 \\ -x^3 + 1, & x < -1 \end{cases}$$

$$\mathbf{f}'(x) = -3x^2$$

Problem 7.1:

Answer: The function is not differentiable because it is not continuous.