Properties of Parabolas

Problem 1 – Focus and directrix

A parabola may be defined as the locus of all points that are equidistant from a fixed point, called the **focus**, and a fixed line, called the **directrix**.

On page 1.3, drag point *P* along the parabola.

- Describe the relationship between the lengths of $\overline{AP}$ and $\overline{PB}$.

- For a parabola with focus (0, 4) and directrix *y* = –4, what is the distance from the focus to the vertex?

Problem 2 – Constructing a parabola and writing its equation

In the diagram on page 2.2:

- Point *D* lies on a horizontal line *m*
- Point *F* is a free point, and $\overrightarrow{FR}$ is perpendicular to line *m*
- Line *l* is the perpendicular bisector of $\overline{FD}$
- Point *P* lies on line *l*, such that $\overline{PD}$ (not shown) is perpendicular to line *m*
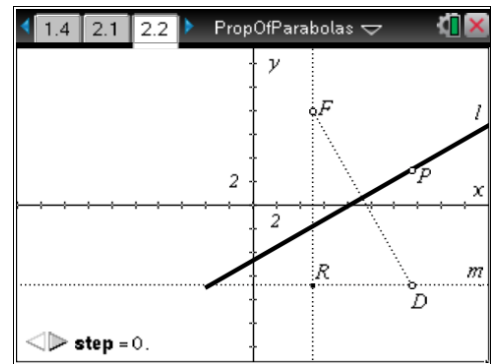
Drag point *D*, and observe the movement of line *l* and point *P*. Use the **Segment** tool to draw $\overline{FP}$ and $\overline{PD}$. Then use the **Length** tool to measure each segment.

- Describe the relationship between the lengths of $\overline{FP}$ and $\overline{PD}$.

Use the **Hide/Show** tool to hide segments $\overline{FD}$, $\overline{FP}$, and $\overline{PD}$ and their measurements. Select the **Locus** tool, click on line *l*, click on point *D*, and then press $\boxed{\text{esc}}$.

- Drag point *D* and describe the shape of the locus of lines.

Select the **Locus** tool once more, click on point *P*, followed by point *D*. When the locus of points appears (as the parabola), use the **Attributes** tool to increase the line weight.

- Drag point *F* vertically and describe changes in the parabola as the focus and directrix become farther apart or closer together.

The equation $y = a(x - h)^2 + k$ describes the parabola with vertex (h, k), where $|a| = \frac{1}{4 \cdot c}$, and *c* is the distance from the focus to the vertex. If *a* is positive, the parabola opens up; if *a* is negative, it opens down.

Hide the locus of lines. Use the **Coordinates and Equations** tool to display and "reset" the coordinates of point *F* to (5, 8).

- The vertex of this parabola is the midpoint of what segment?

Use the **Midpoint** tool to construct the vertex (point *V*), then label its coordinates.

- What are the coordinates of the vertex?

Use the **Length** tool to measure the distance from the focus to the vertex.

- What is the value of *c* for this parabola?

Use the **Text** tool to display the expression $\frac{1}{4 \cdot c}$, and then use the **Calculate** tool to evaluate the expression.

- What is the value of *a* for this parabola?

- Write the equation for this parabola. Graph the equation as **f1**(*x*) to check. Sketch the parabola in the screenshot to the right.
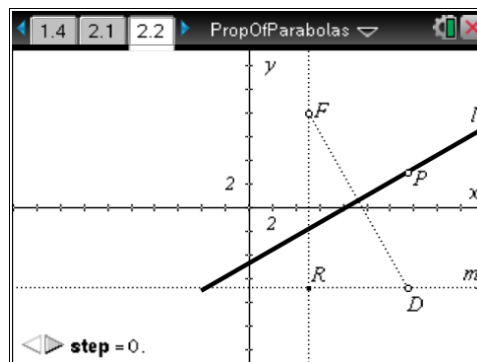
Problem 3 – Focal width and the value of |a|

On page 3.2, a parabola is shown, along with its focus, vertex, and directrix. The dashed segment passes through the focus and is parallel to the directrix. The length of this segment is called the *focal width* of the parabola.

To answer the following question, drag points *F* and *D* to examine different parabolas. You may also use the **Length** tool to find any necessary measurements.

- For a parabola with equation $y = a(x - h)^2 + k$, what is the relationship between the focal width and the value of |a|?