

Exponential Decay

Teacher Notes & Answers

7 8 9 10 **11** 12



Introduction

The most tasteful way to understand exponential functions is through M&M'S®. The concepts covered in this investigation can be applied to a large range of other applications that cover exponential growth and decay. The key here is to understand what it means to be exponential.



The M&M'S packaging warns that **M&M'S may contain traces of nuts**. Always check for food allergies prior to including them in any activity. They have been selected for this activity as a means of creating a more memorable activity.

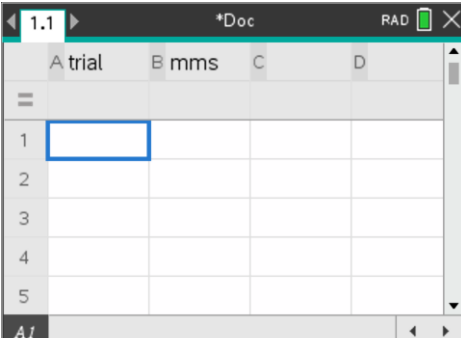
Teacher Notes: Students may be grouped in pairs to reduce the per-student consumption (and cost) of M&M'S. An effective quantity for the experiment is approximately 150 M&M'S which will generally be consumed within 10 trials. Later in the activity students use simulations which include larger initial populations. One advantage of a random quantity of M&M'S at the start of the activity, combined with the randomness of the consumption is that each student (team) will have a different set of data which is a good discussion point with regards to 'modelling' rather than 'exact' results.

Set up

Start a new TI-Nspire document and insert a Lists and Spreadsheet Application.

Column names: "trial" & "mms"

These columns will be used to store the data for the M&M'S experiment.



	A trial	B mms	C	D
1				
2				
3				
4				
5				

Collecting the data:

- Purchase a pack of M&M'S, then pour out all the M&M'S onto a clean surface and count them. The initial trial is referred to as: "Trial 0", record a 0 in cell A1 of the "Trials" column. The initial quantity (count) is stored in cell B1 of the "mms" column.

Note: Check to make sure all the M&M'S have the M symbol on them, some colours may be difficult to read. If any of them do not have the logo, they should be eaten immediately and removed from the initial count.

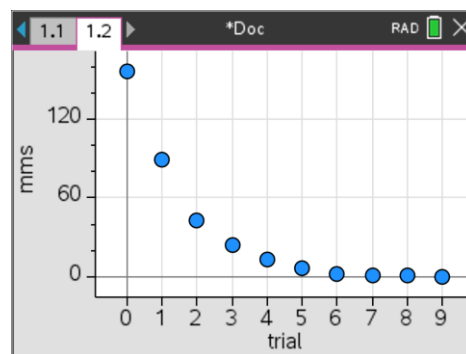
- Put all the M&M'S into a container and shake them up. Pour them out onto the clean surface and eat all the M&M'S that have the M logo facing upwards.
- Count the remaining M&M'S and record the result in cell B2 of the "mms" column. This is trial 1, so record a '1' in cell A2 of the "trials" column.
- Repeat this process until all the M&M'S are consumed, record the trial number and remaining quantity each time.



Graphing the data:

Insert a Data & Statistics Application and graph: trials vs mms.

Sample data shown opposite.

**Determining the Exponential Equation:****Question 1.**

- a) Assume each M&M has an equal probability of being removed and that probability is $\frac{1}{2}$. If there are 'a' M&M'S at the start of the investigation the expected number of M&M'S after one trial will be: $a \times \underline{\quad} = \underline{\quad}$

Answer: $a \times \frac{1}{2} = \frac{a}{2} = a \times 2^{-1}$

- b) The expected number of M&M'S remaining after two trials will be: $a \times \underline{\quad} \times \underline{\quad} = \underline{\quad}$

Answer: $a \times \frac{1}{2} \times \frac{1}{2} = \frac{a}{4} = a \times 2^{-2}$

- c) The expected number of M&M'S remaining after three trials will be: $a \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = \underline{\quad}$

Answer: $a \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{a}{8} = a \times 2^{-3}$

- d) What is the ratio between successive quantities of M&M'S?

Answer: $\frac{1}{2}$ **NOTE:** The purpose of this question is to emphasis the notion of 'ratio'.

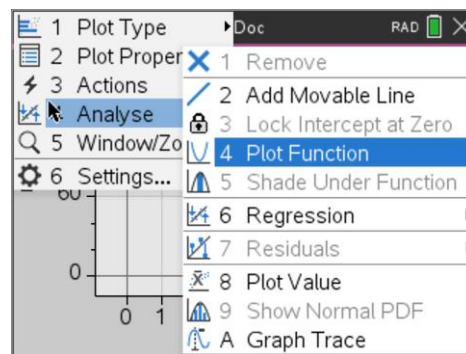
- e) The expected number of M&M'S remaining after x trials will be: $a \times \underline{\quad}$

Answer: $a \times \left(\frac{1}{2}\right)^x = a \times 2^{-x}$

- f) Let y represent the number of M&M'S and x the number of trials. Write down the equation for the expected number of M&M'S in your experiment.

Answer: Answers will vary depending on the initial term. Sample: $N = N_0 2^{-x}$

You can check how well your equation models the data by using the **Plot Function** option in the Analyse menu.



Question 2.

Discuss how well your equation models your data.

Answer: Answers will vary but generally students should note that the equation provides a reasonable model for their data.

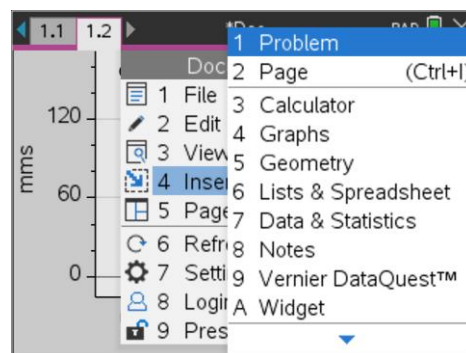
Calculator Simulation:

Rather than consuming another bag of M&M'S, a simulation can be generated using the calculator.

Insert a New Problem: **doc** > **Insert** > **Problem**

Insert a **Lists & Spreadsheet** application.

Notice that the page numbering has changed to 2.1. This means you are now in Problem 2, Page 1. The data you collected for the original experiment is still in Problem 1, but 'invisible' to Problem 2.



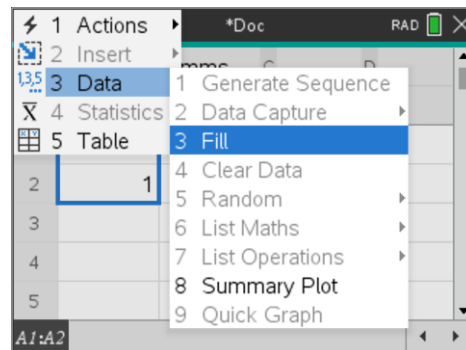
Add the same titles as before for this new spreadsheet.

In the Trials column (column A) insert a 0 in cell A1 and a 1 in cell A2.

Use the shift and arrow keys to select and highlight cells A1 and A2.

Press: **menu** > **Data** > **Fill**

Arrow down to cell A8 and press **enter** the column will automatically populate with the numbers 0, 1, 2, ... 7

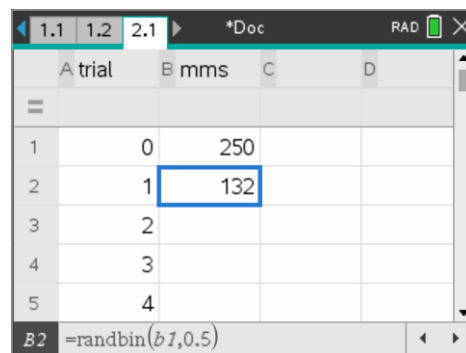


In cell B1 ("mms" column) enter the value: 250. This is our new start value for the number of imaginary MMs in our container.

In cell B2 enter the formula: **=randBin(B1,0.5)**

Note that your value in cell B2 will most likely be different than the one displayed opposite.

Select cell B2 and use the **Fill** command to extend this formula down to cell B8. The formula in cell B3 will refer to the previous quantity in cell B2, the formula in cell B4 will reference cell B3 and so on; this is called 'relative referencing'.

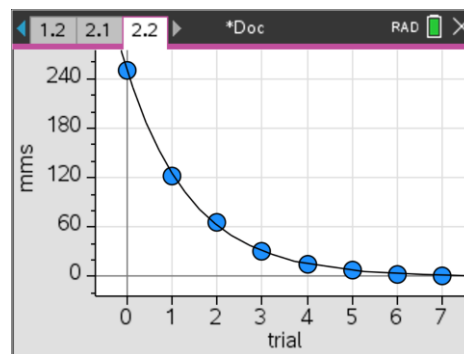


Graph your results using the Data & Statistics Application and insert a function that models the data, sample shown opposite.

Once you have graphed and modelled the data (equation), return to Page 2.1 and press **[ctrl]+R**, this will **Recalculate** the simulation.

Return to the Data and Statistics Application and determine if your equation is still a good fit.

Note: If the simulation produces a '0' (zero) an error may occur, confirm and then generate another simulation.



Question 3:

What is the equation that best represents a model for the simulation data?

Answer: $y = 250 \times 2^{-x}$

Question 4:

Edit the Spreadsheet so that it simulates 500 M&M'S, adjust and record your equation.

Answer: $y = 500 \times 2^{-x}$

Question 5:

Suppose the probability of the M&M logo appearing upright is actually 0.4. Would you expect it to take more or less trials to consume all the M&M'S?

Answer: It would take longer to consume all the M&M'S. $y = 250 \times 0.6^{-x}$

Question 6:

Adjust the simulation to explore Question 5 and write down your equation.

Answer: $y = 250 \times 0.6^{-x}$ **Note:** Simulation: Cell B1 = 500, Cell B2 = RandBin(B1,0.6). 40% probability of M facing upwards means that 60% remain. So with each successive event there will be 60% (0.6) of the previous quantity still present.

Question 7:

Suppose you have a cup with 500 M&M'S. You notice that 50 of the M&M'S do not have the logo. Assuming there is a 50:50 chance of the logo facing upwards:

- Create a simulation for these conditions
- Determine (and record) an equation that models the data.

Answer: Simulation formulas: Cell B1 = 500, Cell B2 = RandBin(B1-50,0.5). Equation: $y = 450 \times 2^{-x} + 50$

Question 8:

Suppose you have 450 M&M'S in your cup. The probability of the M&M logo appearing uppermost is 0.45 and 50 of the M&M's have no logo.

- Create a simulation for these conditions
- Determine (and record) an equation that models the data
- After how many trials are there likely to be fewer than 100 M&M'S remaining?
You can use the 'solve' command to determine the answer, but discuss how this answer might be obtained 'by hand' without using the solve command.

Answer: Simulation formulas: Cell B1 = 450, Cell B2 = RandBin(B1-50,0.55). Equation: $y = 450 \times 0.65^x + 50$. It will take approximately 4 & 5 trials for the number of M&M'S to reach 100.