Activity 3

Exploring the Vertex Form of the Quadratic Function

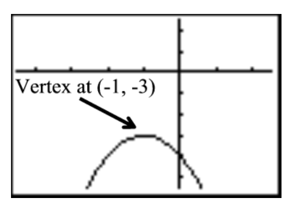
Any quadratic function (parabola) can be expressed in $y = a(x - h)^2 + k$ form. This form of the quadratic function (parabola) is known as the *vertex form*. In this activity, you will discover how this form shows both the location of the vertex and the width and direction of the curve. As you make discoveries, record your findings on a piece of paper for later practice sessions.

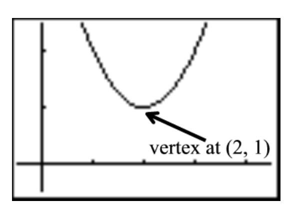
At the end of this activity, you should be able to locate the vertex of any parabola that is expressed in vertex form and determine the direction it opens.

Since you will be able to find the vertex and tell the direction a parabola opens, you will be able to find the maximum or minimum value of quadratic functions expressed in vertex form.

You will end this activity with a beginning knowledge of the general concept of translation and be able to apply it to new situations.

The Graph of a Quadratic Function in Vertex Form





Quadratic functions come in two basic shapes: those that open up and those that open down. One of the important points in a quadratic function is its vertex. The *vertex* is the lowest point (minimum value) if the function opens up and it is the highest point (maximum value) when it opens down.

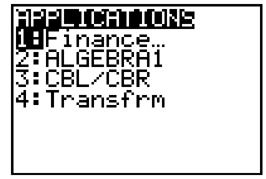
Using your TI-83 Plus, graph $y = (x - 2)^2 + 1$.

Questions for Discussion

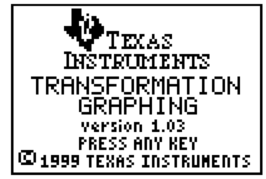
1. Where is its vertex? What direction does it open?

Studying the Effect of A, B, and C

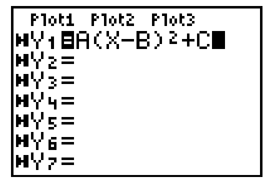
1. Press [APPS] and select Transformation Graphing by pressing the number at the left of **Transfrm.** Press any key (except [2nd] or [ALPHA]) to start Transformation Graphing.



Note: If you do not see the screen illustrated at the right, select ***2:Continue****.*



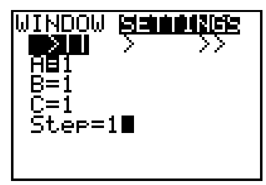
2. In **Func** mode, press [Y=] to display the **Y=** editor. Clear any functions that are listed and turn off any plots. Enter the general vertex form of the quadratic function, $Y = A(X - B)^2 + C$. Press [ALPHA] **A** [(] [X,T,Θ,n] [−] [ALPHA] **B** [)] [x^2] [+] [ALPHA] **C.**



   If the Play-Pause Mode is not selected at the left of **Y1** (>||), press [◄] until the cursor is over the symbol and then press [ENTER] until the correct symbol is selected.

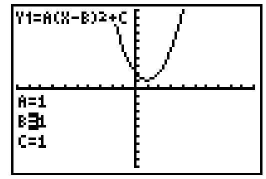
   Note: You entered $Y=A(X - B)^2 + C$ in place of $Y=A(X - H)^2 + K$, which is the form commonly found in textbooks, because Transformation Graphing only uses the coefficients A, B, C, and D.

3. Press [WINDOW] [▲] to display the settings screen for Transformation Graphing.



   As a starting place, set the SETTINGS as pictured. To make these selections, press [▼] **1** [▼] **1** [▼] **1** [▼] **1.** This defines the starting values for the coefficients and the increment by which you want to observe the change in the coefficients.

Studying the Effect of B



4. Press [ZOOM] **6:ZStandard** to display the graph. The graph will show the pre-selected values of **A**, **B**, and **C**. Both the X and Y-axis range from -10 to 10 with a scale of 1.

   Press [▼] to move down one space and highlight **B=**. You will start your study with the effect of **B**.

5. Press [▶] to increase the value of **B** by the pre-selected Step value (1 in this example). The graph is automatically redrawn showing the effect of this change on **B**. Continue to press [▶] until you have an idea of how changing **B** effects the graph.

6. Press [◀] to decrease the value of **B** by the pre-selected Step value. Did the graph move the direction you would have expected?

Questions for Discussion

1. Changing the value of **B** has the curve move in what direction?

   This moving of the curve is called a translation in the X-direction or horizontal translation.

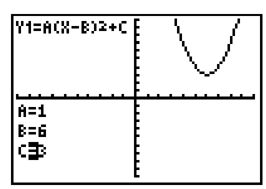
2. Use the cursor keys ([◀] [▶]) to change the value of **B** again. As you change **B** notice the x-coordinate of the vertex.

   If **B=3**, where is the vertex? How about **B=5**? And when **B=-2**?

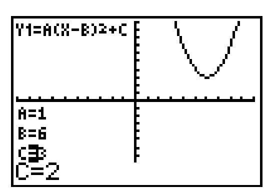
Make a hypothesis about the relationship between **B** and the vertex of the parabola. Test your hypothesis by entering **B=1**, **B=3**, **B=5**, **B=-1**, and **B=-2**. Were you correct?

In vertex form $Y = A(X - B)^2 + C$, the value of **B** gives the x-coordinate of the vertex. Be careful, though. Notice the form has X - B. If you had the equation $Y=(X - 3)^2$ then B=3 and the vertex is at X = 3. For the equation $Y=(X + 1)^2$ the **B** would be **-1** with the vertex at X = -1.

Studying the Effect of C



1. Press [▼] to highlight the **C=**. Press [▶] several times and notice the change in the graph. Press [◀] several times and notice this change.

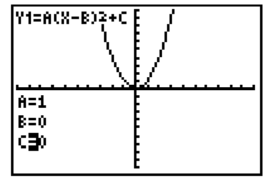


2. Predict where the vertex of the function will be if you let **C=2**. Enter **2** for **C** and check your prediction.

3. Make some conclusions about the effect of changes in **C** on the vertex. Check your conclusions by entering test values for **C**.

Changes in **C** create a vertical translation of the curve. When **C** increases the curve moves up. When **C** decreases the curve moves down. The value of **C** is the y-coordinate of the vertex.

Check Your Understanding So Far



The graph of $y = x^2$ is shown at the right. The following equations are vertical and horizontal translation of $y = x^2$. Use what you have discovered about translation of the vertex of a quadratic function to predict the vertex of the graph of each equation. Check your prediction using Transformation Graphing.

$y = (x - 2)^2$

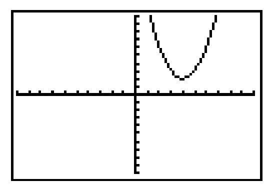
$y = (x - 2)^2 + 3$

$y = (x + 1)^2 +3$

$y = (x + 1)^2 -2$

$y = (x + 5)^2$

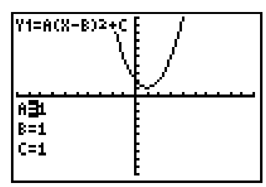
$y = x^2 - 2$



What is the equation of the parabola (quadratic function) graphed at the right?

Note: The scale is 1.

Studying the Effect of A



1. Return to the Transformation Graphing Screen and press ▲ until the **A=** is highlighted.

2. Use the same discovery method you used with **B** and **C** to investigate the effect of **A** on the graph of the parabola. Be sure to let **A** be both negative and positive. Once you have a hypothesis and have checked it for the effect of **A**, continue with the next question.

Questions for Discussion

1. What effect does changing the value of **A** have on the graph? Be sure to discuss both magnitude and sign changes.

The value of **A** determines the direction of the parabola and its width. The larger the magnitude of **A**, the narrower the curve. The smaller the magnitude of **A**, the wider the curve. A positive sign means the parabola is opening up. A negative sign means the parabola is opening down.

Deactivate Transformation Graphing before continuing.

1. Press [APPS] and select the number preceding **Transfrm**.

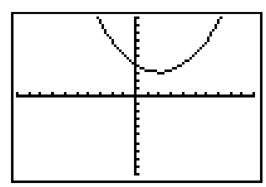
2. Select **1:Uninstall**.

Check Your Understanding

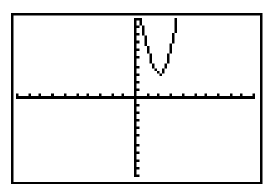
Match the equation from column 1 with its graph in column 2. Be careful that you look at all the equations and compare them before you answer any questions. Do these first without using your calculator, and then verify your answers using your calculator.

Note: These examples only investigate ***A****.*

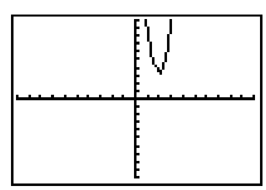
1. $Y = 3(X - 2)^2 + 3$

a. 

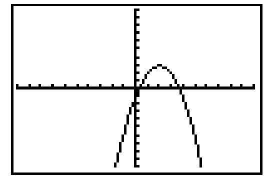
2. $Y = -(X - 2)^2 + 3$

b. 

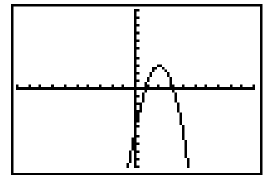
3. $Y = .25(X - 2)^2 + 3$

c. 

4. $Y = -2(X - 2)^2 + 3$

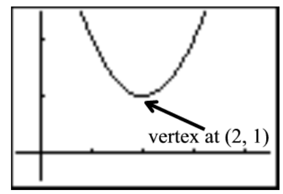
d. 

5. $Y = 6(X - 2)^2 + 3$

e. 

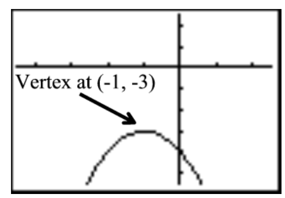
Maximum and Minimum Values Come Into Focus

When a parabola opens upward, the vertex will be the lowest point on the curve. Any other point will have a larger value for *y*. In the graph at the left, the y-value of the vertex is 1. This is the lowest value *y* can obtain and it is thus called the *minimum* value of the function.



The graph shows a parabola, quadratic function, with a minimum value of 1 when **x=2**.

Likewise, when a parabola opens down there will be a largest or maximum value for *y*. This graph shows a function with a maximum value of -3 when **x= -1**.



Check This Out

Complete the table for each parabola.

Equation	Opens up/down	Function has a maximum/ minimum	Maximum/ minimum value
$Y = 2(x - 3)^2 + 2$	Up	minimum	2
$Y = -3(x + 1)^2 + 10$			
$Y = 10(x + 4)^2 - 36$			
$Y = -16(x - 2)^2 - 100$			

A Quick Application

The equation $y = -16(x - 4)^2 + 259$ models the flight of a model rocket where *y* is the height of the rocket and *x* is the time since it was launched. Graph the function with a reasonable domain. What is the maximum height of the rocket? How long after it was launched did it reach its maximum? What does this have to do with this activity?

Homework Page

Name ____________________

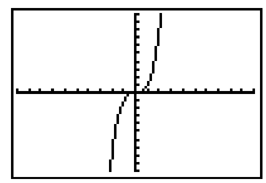
Date ____________________

Look at some equations of linear functions and see how translation applies.

1. Use your graphing calculator to graph **Y1 = X** and **Y2 = X + 3** on the same axis. In what two ways is the second equation a translation of the first?

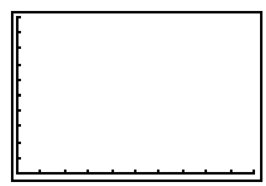
Now let's look at some functions you might not have already studied and see if you can apply your knowledge in a new situation.

2. The graph of the function $\mathbf{y = x^3}$ goes through the origin (0, 0). Look at the graph of $\mathbf{y = x^3}$ at the right and using the point at the origin as the point you translate (like you did the vertex) sketch the graph of $\mathbf{y = x^3 + 2}$. Check your answer by graphing $\mathbf{y = x^3 + 2}$ on your TI-83 Plus.

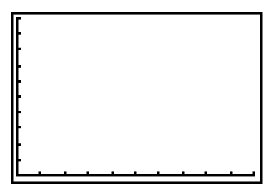


Note: You can either use [^] **3** *for the power of three or* [MATH] ***3:*** ^{3}.

3. Sketch $\mathbf{y = (x - 2)^3}$ and check your answer with your graphing calculator.



4. Sketch $\mathbf{y = (x + 1)^3 - 5}$ and check your answer.



Notes for Teachers

The purpose of this activity is to allow the students to "play" with the vertex form of the parabola and discover how the vertex, direction, and width can be determined by looking at the parameters.

Once the activity is complete, the students should be very capable of making predictions about the location of the vertex of a parabola expressed in vertex form. They will need much more work with the effect of **A**. After this activity, they will only be capable of comparing two graphs; they will not be able to graph a parabola in vertex form by hand. Students will need classroom instruction to graph the actual width.

The concept of translation is basic to the general study of functions, and the activity ends with a quick look at translation in general. These closing questions on translation are not really part of the study of the vertex form; you can easily leave them out. They serve as a quick view of translation for those who use translation as part of their study of function.

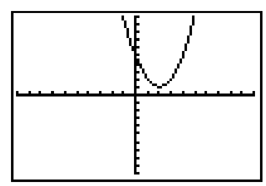
The work with translation, at the end, encourages the students to make a hypothesis and use their calculator to check it.

The skills and knowledge developed in this activity will be used in the next activity.

Answers

The Graph of a Quadratic Function in Vertex Form: Questions for Discussion

1. Vertex is at (2, 1). The parabola opens up.



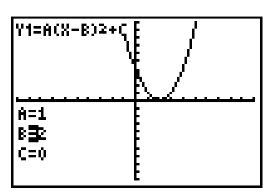
Studying the Effect of B: Questions for Discussion

1. The curve moves in the X-direction.
2. When **B=3**, the vertex is at -3. **B=5** gives a vertex of -5. **B=-1** gives a vertex at 1. The vertex is 1 if **B=-1**.

Check Your Understanding So Far

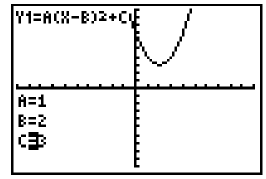
$y = (x - 2)^2$

Vertex is at (2, 0)



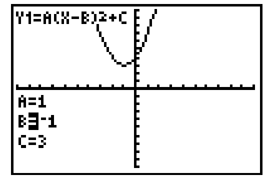
$y = (x - 2)^2 + 3$

Vertex is at (2, 3)



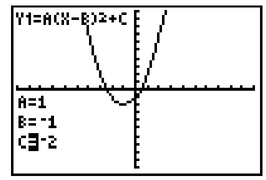
$y = (x + 1)^2 +3$

Vertex is at (-1, 3)



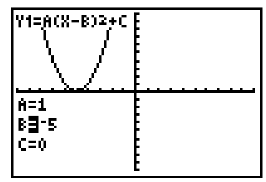
$y = (x + 1)^2 -2$

Vertex is at (-1, -2)



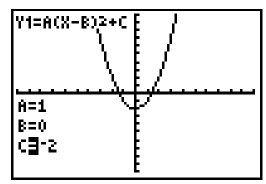
$y = (x + 5)^2$

Vertex is at (-5, 0)



$y = x^2 - 2$

Vertex is at (0, -2)



What is the equation of the parabola (quadratic equation function)?

$Y = (x - 4)^2 + 2$

Studying the Effect of A: Questions for Discussion

1. The value of **A** determines the direction of the parabola and its width. The larger the magnitude of **A**, the narrower the curve. The smaller the magnitude of **A**, the wider the curve. A positive sign means the parabola is opening up. A negative sign means the parabola is opening down.

Check Your Understanding

1. b
2. d
3. a
4. e
5. c

Check This Out

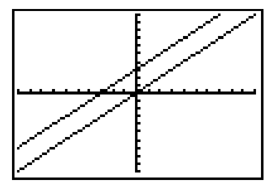
Equation	Opens up/down	Function has a maximum/minimum	Maximum/minimum value
$Y = 2(x - 3)^2 + 2$	Up	Minimum	2
$Y = -3(x + 1)^2 + 10$	Down	Maximum	10
$Y = 10(x + 4)^2 - 36$	Up	Minimum	-36
$Y = -16(x - 2)^2 - 100$	Down	Maximum	-100

A Quick Application

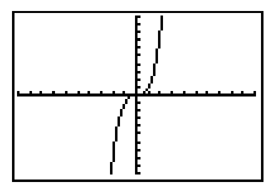
The maximum height is 259 feet. It takes 4 seconds to reach the maximum. The vertex is the maximum or minimum value of the function. Since **A** is negative, the parabola opens down and has a maximum.

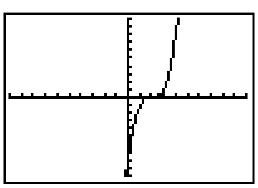
Homework Page

1. The second function *($y = x + 3$)* can be viewed either as a translation 3 units up or a translation 3 units to the left. If it is viewed as a translation 3 units up, you would be viewing the equation as $y = x + 3$. If you view it as a translation 3 units to the left (x-direction), you would be seeing the equation as $y = (x + 3)$.



2. The graph of $y = x^3 + 2$ is a vertical translation of $y = x^3$ two units up. The point at the origin should be moved to (0, 2).



3. 

4.