

According to the Standards:

**Instructional programs from preK-grade 12 should enable students to:**

- Recognize and use connections among mathematical ideas
- Use the language of mathematics to express mathematical ideas precisely
- Select, apply and translate among mathematical representations to solve problems

**In grades 9-12 students should**

1. Students should develop an increased capacity to link mathematical ideas and a deeper understanding of how more than one approach to the same problem can lead to equivalent results.

**Calculus Scope and Sequence:** Applications of Derivatives

**Keywords:** optimization, maximum, minimum, applications

**Description:** This activity will illustrate the idea of using the derivative to find a solution to an optimization problem

*A farmer wishes to fence in a rectangular field of 10,000 square feet. The north-south fences will cost \$1.50 per foot, while the east-west fences will cost \$6.00 per foot. Find the dimensions of the fence that will minimize the cost.*

- Set up the cost function in terms of one variable
- Find the derivative of the cost function
- Find the critical points on the derivative, check for max, min using the second derivative
- Confirm the point graphically.

The Derivative is found from the Homescreen in F3-Calc-#1

Syntax:  $d(\text{function, variable})$

The Solve function is found from the Homescreen in F2-Algebra-#1

Syntax:  $\text{solve}(\text{expression} = \text{expression, variable})$

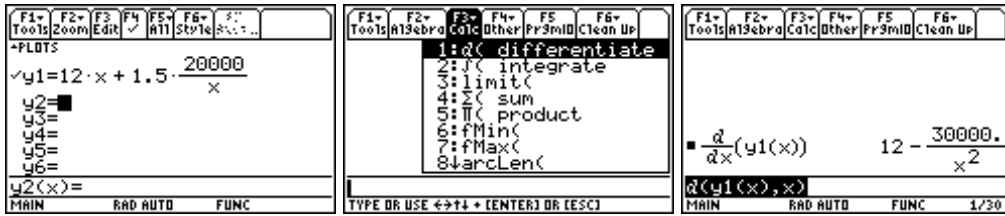
**User Tips:**

1. You can store the function in the Y= screen to make it easier to use
2. You can copy and paste a result from the homescreen by using the Up Arrow to highlight it and then pressing ENTER to paste it into the edit line. (You can also use the copy & paste functions in the F1-Tools menu)

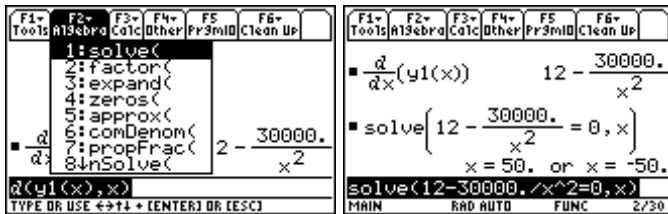
In this problem we let  $x$  = east-west dimension and  $y$  = north-south dimension

- Therefore  $xy = 10,000$
- $y = 10,000/x$
- $Cost = 6(\text{east-west}) + 1.5(\text{north-south})$
- $Cost = 6(2x) + 1.5(2y)$
- $Cost = 12x + 1.5(20,000/x)$

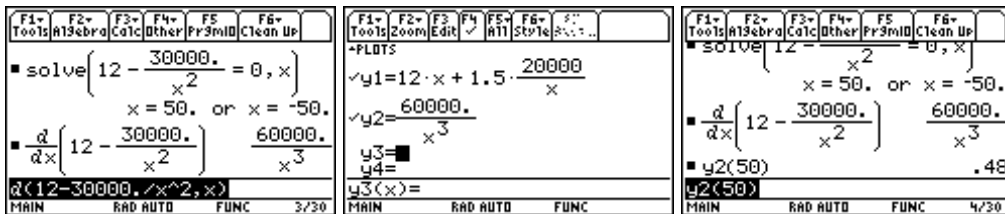
Finding the Derivative:



Setting the Derivative equal to zero to find the critical points:



Disregard  $-50$  (outside the domain of the problem) and check in the second derivative:

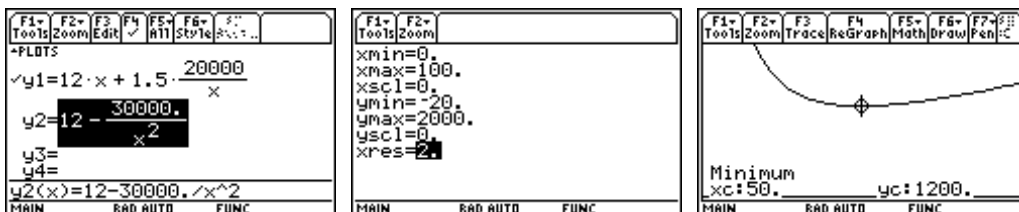


The second derivative is positive, therefore the function has a minimum at  $x=50$  in the domain  $x > 0$  which is appropriate for the problem.

The answer to the question asked then is 50 feet of east-west fence and 200 feet of north-south fence.

You can use the graph to visually confirm these dimensions:

**Note: Make sure only the cost equation is active, and that you find a reasonable window to see the problem develop.**



**Extension:**

You can show the graphs of the original cost function and the first derivative and compare. However, the window setting required to show both does make it hard to see.

