Chi Square Distributions

TIMATH.COM: STATISTICS

TEACHER NOTES

ID: 9737

Activity Overview

In this activity, students first compare the chi-square distribution to the standard normal distribution (for five degrees of freedom). They will also determine how the Chi-Square distribution changes as they increase the degrees of freedom. Students then confirm critical values of χ^2 for given confidence levels and sample sizes they find in a chart and finish the activity by constructing confidence intervals for real-life scenarios.

Topic: Continuous Distributions

- Variance and Standard Deviation
- Degrees of Freedom
- Critical Values and Confidence Intervals

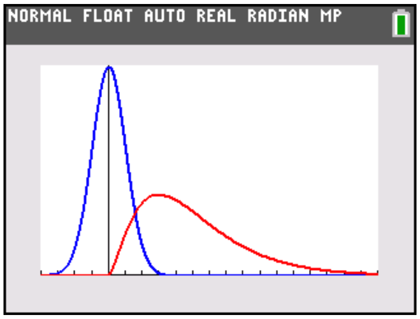
Teacher Preparation and Notes

- Students should already be familiar with the normal distribution and it characteristics, as well as finding and interpreting confidence intervals for normal distributions.
- Using a confidence interval to make a decision, as done in Problem 3, is a precursor to hypothesis testing.
- Students will need to have access to a chart of chi-square distribution critical values. You can either have them look in a Statistics textbook, or you can copy a chart from a textbook to include with the student worksheets. If needed, review how to use the chart of critical values before beginning Problem 2.
- **To download the student worksheet, go to education.ti.com/exchange and enter "11599" in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- F Distribution (TI-84 Plus family) — 9780
- Is it Rare? (TI-84 Plus family) — 9094
- Candy Pieces (TI-84 Plus family) — 10039
- Cancer Cluster (TI-Nspire™ technology) — 9996



This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- ChiSquareDistributions_Student.pdf
- ChiSquareDistributions_Student.doc

Click HERE for Graphing Calculator Tutorials.

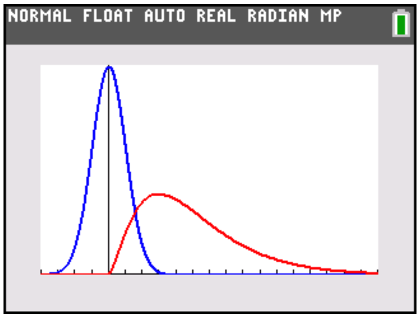
Problem 1 – Characteristics of the Chi-Square Distribution

The student worksheet contains the following introduction on the Chi-Square distribution. Discuss this with students before beginning the activity.

> The chi-square distribution is a distribution of sample variances (s^2). It has two critical values which are used when constructing confidence intervals to estimate the population variance (σ^2). The number of degrees of freedom is one less than the sample size. The chi-square distribution requires that samples be taken from a normally distributed population.

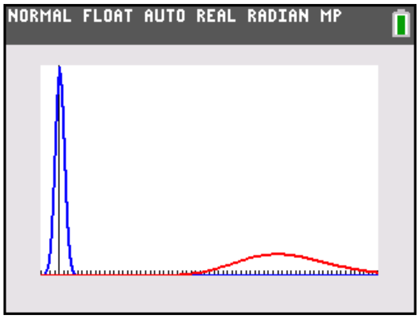
Students will first graph the standard normal distribution and a chi-square distribution with 5 degrees of freedom. They should see that the chi-square distribution is skewed to the right and none of the values are negative.

To distinguish the graphs, students can change the thickness of the line by pressing enter to the left of **Y2** in the [Y=] screen.



Students will then view the graph as they increase the degrees of freedom of the chi-square distribution. They will need to change the **Xmax** value of the viewing window (the Xmin, Ymin, and Ymax can stay the same).

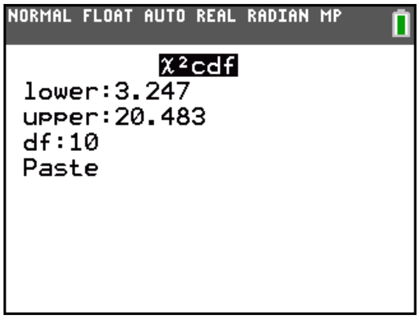
They should see that as the number of degrees of freedom increases, the distribution becomes less skewed and more symmetric. It appears that the mean equals the number of degrees of freedom.



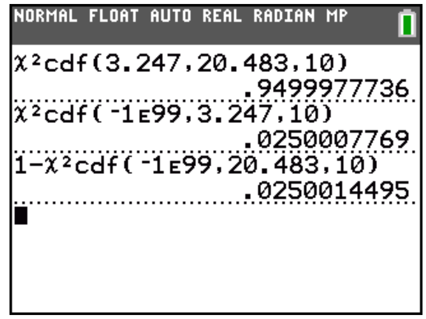
Problem 2 – Critical Values for a Chi-Square Distribution

Students will now need to use a Chi-Squared chart to find the critical values for a 95% confidence interval, for n = 11.

To check the values, students will enter the command shown at the right, which finds the area between the two critical values.



The commands shown at the right show how students can check that 2.5% is in the left tail and 2.5% is in the right tail.



Students are to compare the critical values for n = 12 for the 80% and 90% levels and explain why, at the 90% level, the left value is smaller and the right value greater.

(greater confidence = more room for error)

Problem 3 – Constructing a Confidence Interval

Students are given the following formulas for constructing confidence intervals on the worksheet. When introducing the formulas, be sure that students notice χ^2_L is on the right and χ^2_R is on the left.

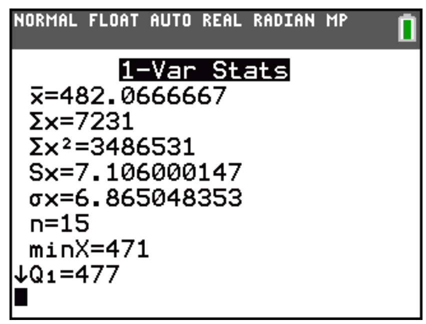
Confidence Interval for Population Variance

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

Confidence Interval for Population Standard Deviation

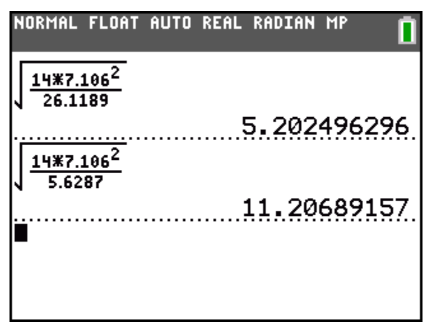
$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

Students are to enter the weights given on the worksheet into List **L1**. If there is already data in the list, have students move the cursor to the top of L1 and press [CLEAR] [ENTER].



Using the **1-Var Stats** command, students can find the sample standard deviation.

Students will need to use the chart to find the critical values and then use those to calculate the confidence interval of the population standard deviation.



Students can press [ALPHA] [F1] and select **n/d** to insert the fraction template.

We are 95% confident that the true standard deviation lies between 5.2025 and 11.2069. Since 6 is in the interval, the weights are considered consistent, that is, the differences are acceptable.

Point out that if the standard deviation had to be less than or equal to 5 grams, then the differences would not be considered acceptable because 5 is outside the interval.

Problem 4 – Practice

Have students work though the questions in Problem 4 on their own. When everyone is finished, review the answers and answer any questions.

Solutions

1. The chi-square distribution is skewed to the right and none of the values are negative.

2. As the number of degrees of freedom increases, the distribution becomes less skewed and more symmetric. It appears that the mean equals the number of degrees of freedom.

3. 3.247 and 20.483

4. about 0.949998; Note that how close this value is to 0.95 depends on how many decimal places are listed in the charts that students use.

5. Use –1E99 for the lower bound, ${\chi^2}_R$ for the upper bound, and subtract the answer from 1.

6. Sample: The left value for the 90% level will be smaller than the one for the 80% level and the right value for the 90% level will be greater than the one for the 80% level. In essence, the area between the tails will spread because as confidence increases, so does the margin of error.

7. 80%: 5.578 and 17.275; 90%: 4.575 and 19.675

8. about 7.106

9. from 5.2025 to 11.2069

10. We are 95% confident that the true standard deviation lies between 5.2025 grams and 11.2069 grams. Because the standard deviation can be up to 6 grams and 6 is in the confidence interval, the weights are considered consistent, the differences are acceptable.

11. No, 5 grams is outside the confidence interval.

12. 1.852 h to 3.352 h

13. 80%: 24.003 to 34.185
90%: 22.972 to 36.202