Bell Ringer: The Radioactive Decay Equations – ID: 13772

Time required
15 minutes

Topic: Nuclear Physics

- *Write equations for half-life and radioactive decay.*

Activity Overview

In this activity, students derive the equation for the number of parent atoms remaining as a function of time. They rearrange the equation to find the equation for half-life.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire CAS^{TM} technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Calculator, Notes

Teacher Preparation

Before carrying out this activity, review with students exponential equations.

- *This activity is designed to be completed using TI-Nspire CAS technology. If your students do not have TI-Nspire CAS technology, you can give them the equation for N as a function of N_0, k, and t.*

- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on pages 6 and 7 for a preview of the student TI-Nspire document (.tns file). The solution .tns file contains sample responses to the questions posed in the student .tns file.*

- ***To download the student .tns file and solution .tns file, go to education.ti.com/exchange and enter "13772" in the search box.***

- *This activity is related to activity 9522: Nuclear Decay and Chain Reactions. If you wish, you may extend this bell-ringer activity with the longer activity. You can download the files for activity 9522 at education.ti.com/exchange.*

Classroom Management

- *This activity is designed to be **teacher-led**, with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*

- *If you wish, you may modify this document for use as a student instruction sheet. You may also wish to use an overhead projector and TI-Nspire computer software to demonstrate the use of the TI-Nspire to students.*

- *If students do not have sufficient time to complete the main questions, they may also be completed as homework.*

- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following question will guide student exploration during this activity:
- What equations describe radioactive decay?

Students will use TI-Nspire CAS technology to derive an equation for the number of parent atoms present at a given time. They will then determine the equation for half-life and use it to solve several problems.

Step 1: Students should open the file **PhysBR_week30_decay_eqs.tns** and read the first four pages. If you wish, review integrals and derivatives with students. Then, students should move to page 1.5, which gives instructions on how to use the *deSolve* function to find the solution of a differential equation. (Students can press (ctrl)(tab) to move between applications on the page.) The *deSolve* function has the following syntax:

deSolve(*ODE* and *init_cond, var*, *depvar*)

where *ODE* is the differential equation to be solved, *init_cond* is the initial conditions, *var* is the independent variable, and *depvar* is the dependent variable. For the radioactive decay equation, the initial condition is that $N(0) = N_0$. The independent variable is time, t, and the dependent variable is the number of parent atoms, N. In the *deSolve* function, the differential equation must be written using prime symbols to represent derivatives (i.e, instead of writing $\frac{dN}{dt} = -kN$, the equation must be written in the form $N' = -kN$). Once students have typed in the equation, they should press (enter) to evaluate it. Then, they should answer question 1.

Q1. What is the functional form of the decay equation?

A. *The equation is exponential: $N = N_0 e^{-kt}$.*

Note that the TI-Nspire automatically makes all variables lowercase. For ease of reading, certain variables are capitalized in this document, although they appear as lowercase in the .tns file. Variable names are not case-sensitive, so students can type in capital or lowercase letters. For all functions in this activity, students can either type in the function using the keypad or they can insert the function from the Catalog by pressing (catalog) and choosing tab 1.
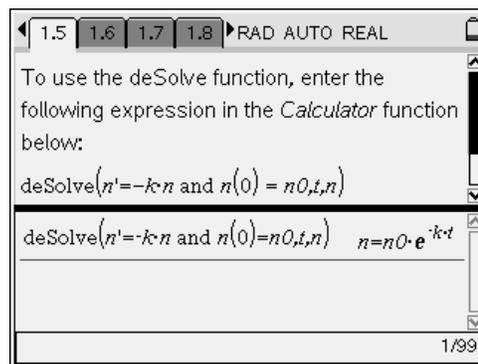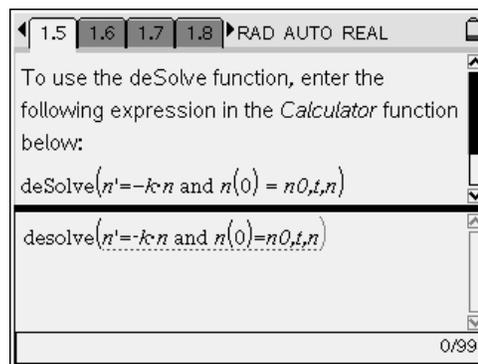
Step 2: Next, students should move back to page 1.5 and assign the decay equation to the name *decay_eq*. To name the equation, students should type *ans →decay_eq* into the *Calculator*. To insert the arrow, students should press (ctrl)(📖) and then choose the arrow from the menu. Alternately, students can press (ctrl)(sto▸var) to insert an arrow. To insert an underscore (_), students should press (ctrl)(⊖). After students have named the equation, they should read pages 1.8 and 1.9, and then move to page 1.10.
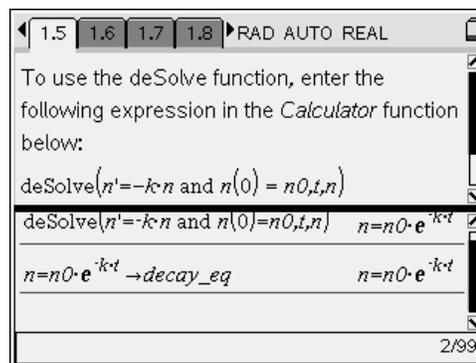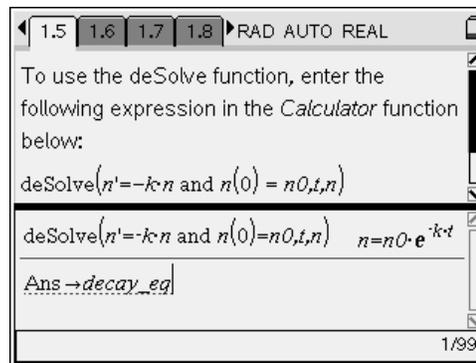
Step 3: On page 1.10, students should use the *Solve* function to rearrange the decay equation to solve for *t*. Students should use the following syntax:

Solve(decay_eq,t) → t_eq

Then, students should answer question 2.

Q2. An important quantity to know for radioactive isotopes is the half-life. Half-life is the time required for one-half of the original parent atoms to decay. After one half-life, what is the ratio of N to N_0?

A. *After one half-life, the number of parent atoms remaining (*N*) is equal to one-half the original number (*N_0*). Therefore, after one half-life,*

$$\frac{N}{N_0} = \frac{1}{2}.$$
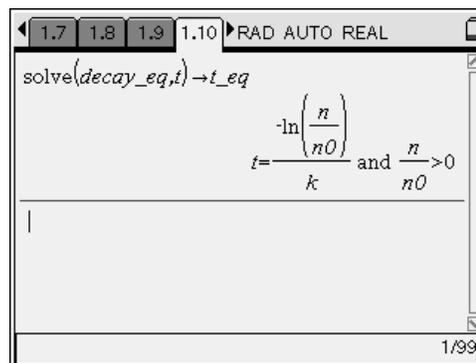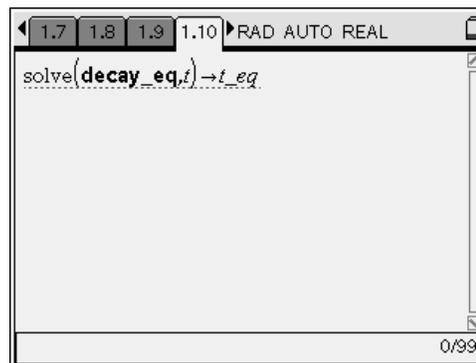
Step 4: Next, students should define the function *t12_eq* by evaluating *t_eq* for the case $N = N_0/2$, using the following syntax:

t12_eq := t_eq | N=N0/2

They should then answer questions 3–7.

Q3. Write the general equation for half-life as a function of decay constant.

   A. *The equation for half-life is* $t_{1/2} = \dfrac{ln(2)}{k}$.
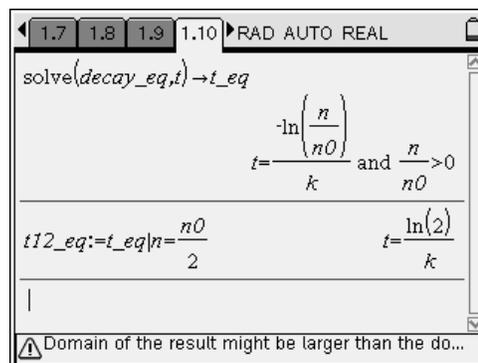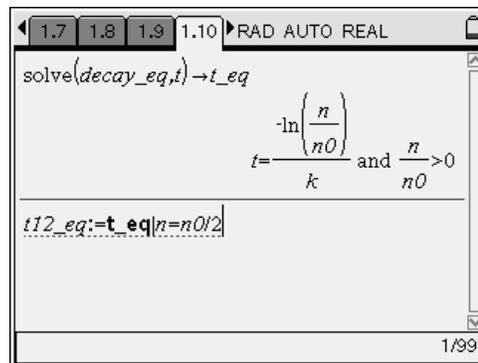
Q4. Potassium-40 has a decay constant of 5.5×10^{-10} yr^{-1}. What is its half-life in years?

   A. *To solve this problem, students should use the following syntax:*
   approx(t12_eq)|k=5.5·10^{-10}
   The approx *command forces the TI-Nspire to return a numerical value. The pipe (|) indicates a temporary value for* k—*i.e., it defines* k *for this evaluation only. The half-life of potassium-40 is approximately 1.26×10^9 yr.*
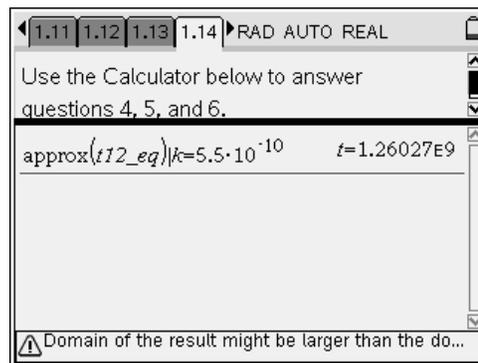
Q5. Rearrange the half-life equation to write a new equation for the decay constant *k* as a function of half-life. Test your answer using the *Solve* function.

   A. *The equation for the decay constant as a function of half-life is* $k = \dfrac{ln(2)}{t_{1/2}}$.

Q6. The half-life of carbon-13 is 5730 yr. What is its decay constant? (Hint: First solve the half-life equation for k.)

A. *To solve the equation for* k *and name the resulting equation* k_eq*, students should enter* solve(t12_eq,k) → k_eq*. They should then use the* approx *command and the pipe, as shown in question 4, to evaluate the equation. The decay constant for carbon-13 is approximately 0.000121 yr^{-1}.*

$approx(t12\_eq)|k=5.5 \cdot 10^{-10}$	$t=1.26027E9$
$solve(t12\_eq,k) \to k\_eq$	$k=\dfrac{\ln(2)}{t}$
$approx(k\_eq)|t=5730$	$k=0.000121$
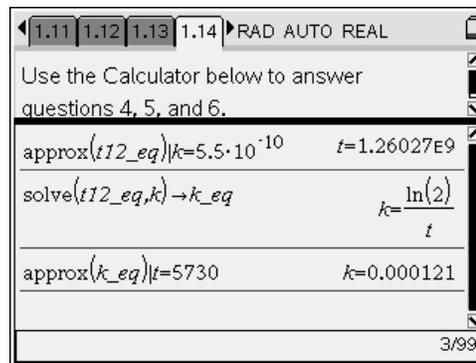
Q7. Helium-3 has a decay constant of 0.000154 d^{-1}. How long is two half-lives of helium-3, in years?

A. *First, students should solve the half-life equation to calculate the half-life of helium-3. Then, they should multiply the half-life by 2 and divide by 365 to calculate the number of years in two half-lives of helium-3. Two half-lives of helium-3 is approximately 24.7 yr.*

$approx(t12\_eq)|k=1.54E-4$	$t=4500.96$
$(t=4500.9557179218) \cdot 2$	$2 \cdot t=9001.91$
$\dfrac{2 \cdot t=9001.9114358436}{365}$	$\dfrac{2 \cdot t}{365}=24.6628$
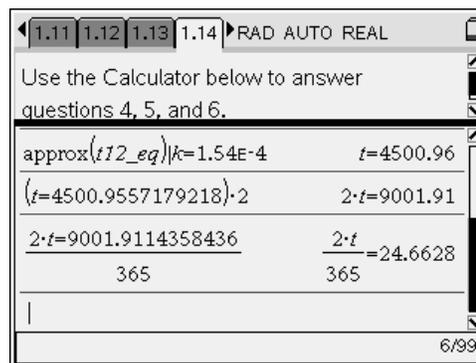
Suggestions for Extension Activities: If you wish, you may have students graph the decay equation (you will need to give them values for N_0 and k). If students have had enough calculus experience, they can solve the differential equations by hand.

Bell Ringer: The Radioactive Decay Equations – ID: 13772

(Student)TI-Nspire File: *PhysBR_week30_decay_eqs.tns*

| 1.1 | 1.2 | 1.3 | 1.4 ▶ RAD AUTO REAL |

THE RADIOACTIVE DECAY EQUATIONS

Physics
Nuclear Physics

| 1.1 | 1.2 | 1.3 | 1.4 ▶ RAD AUTO REAL   ctrl |

Radioactive isotopes decay at a rate that is proportional to the number of atoms present. In other words, the more radioactive isotope atoms there are in a sample, the more decays will happen for each unit of time. The decay rate can be written as an integral:

$\frac{d}{dt}(N) = -kN$, where N is the number of radioactive atoms, t is time, and k is a

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| 1.1 | 1.2 | 1.3 | 1.4 ▶ RAD AUTO REAL |

Every radioactive isotope has its own decay constant, but all isotopes decay according to that same equation. In this activity, you will first determine the equation for the number of radioactive (parent) isotopes as a function of time. Then, you will rearrange the equation to derive the equation for half-life.

| 1.1 | 1.2 | 1.3 | 1.4 ▶ RAD AUTO REAL |

The equation $\frac{d}{dt}(N) = -kN$ is a differential equation. It can be solved by rearranging and integrating both sides:

$$\int\left(\frac{1}{N}\right)dN = \int(-k)dt$$

You can use the TI-Nspire CAS's deSolve function to solve this type of differential equation.

| 1.2 | 1.3 | 1.4 | 1.5 ▶ RAD AUTO REAL |

To use the deSolve function, enter the following expression in the *Calculator* function below:

deSolve$(n' = -k \cdot n$ and $n(0) = n0, t, n)$

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| 1.3 | 1.4 | 1.5 | 1.6 ▶ RAD AUTO REAL |

1. What is the functional form of the decay equation?

| 1.4 | 1.5 | 1.6 | 1.7 ▶ RAD AUTO REAL |

Next, give the equation a name. Go back to page 1.5 and assign the equation to the name *decay_eq*.

| 1.5 | 1.6 | 1.7 | 1.8 ▶ RAD AUTO REAL |

The decay equation allows you to calculate the number of parent isotopes that exist at time t if you know the initial number of isotopes and the decay constant. But suppose you want to know how long it will take for the number of parent isotopes to decay by a certain percentage? In that case, you need to solve for t.

| 1.9 | 1.10 | 1.11 | 1.12 ▶ RAD AUTO REAL |

The Solve function allows you to solve an equation for a variable. The syntax is:

Solve$([eqn],[variable])$

Solve the decay equation for t in the *Calculator* on the next page. Give the rearranged equation the name *t_eq*.

| 1.7 | 1.8 | 1.9 | 1.10 ▶ RAD AUTO REAL |

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| 1.8 | 1.9 | 1.10 | 1.11 ▶ RAD AUTO REAL |

2. An important quantity to know for radioactive isotopes is the half-life. Half-life is the time required for one-half of the original parent atoms to decay. After one half-life, what is the ratio of N to N0?

| 1.9 | 1.10 | 1.11 | 1.12 ▶ RAD AUTO REAL |

In the *Calculator* below, define N to be equal to one-half N0, and then re-evaluate *t12_eq*.

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◀ 1.10 1.11 1.12 1.13 ▶ RAD AUTO REAL

3. Write the general equation for half-life as a function of decay constant.

◀ 1.11 1.12 1.13 1.14 ▶ RAD AUTO REAL

Use the Calculator below to answer questions 4, 5, and 6.

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◀ 1.12 1.13 1.14 1.15 ▶ RAD AUTO REAL

4. Potassium-40 has a decay constant of 5.5×10^{-10} yr^{-1}. What is its half-life in years?

◀ 1.13 1.14 1.15 1.16 ▶ RAD AUTO REAL

5. The half-life of carbon-13 is 5730 yr. What is its decay constant? (Hint: First solve the half-life equation for k.)

◀ 1.14 1.15 1.16 1.17 RAD AUTO REAL

6. Helium-3 has a decay constant of 0.000154 d^{-1}. How long is two half-lives of helium-3, in years?